Strong dissipation feedback: stabilisation of mixed ODE-PDE port-Hamiltonian systems

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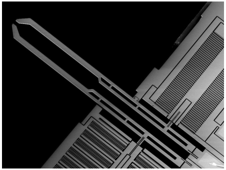
Motivation



1. **Model** that includes the rigid and the flexible dynamics
2. **Control law** that moves the system in the desired configuration and stabilizes the flexible deformations

Motivation

Models' **possible dynamics**:

- Actuator inertia dynamics → ODE
- Rigid dynamics → ODE
- Flexible dynamics → PDE

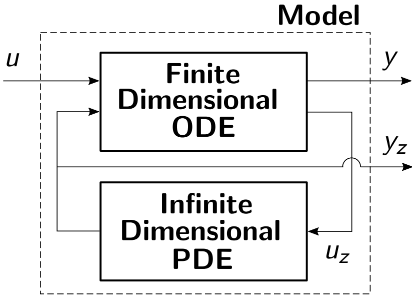


Figure: Model composed by the interconnection between ODE and PDE with **input on the ODE**.

Motivation

Models' **possible dynamics**:

- Actuator inertia dynamics $\rightarrow$ ODE
- Rigid dynamics $\rightarrow$ ODE
- Flexible dynamics $\rightarrow$ PDE

We **restrict** our study on:

- Boundary control systems
- Linear models (Approximation)
- First order PDE

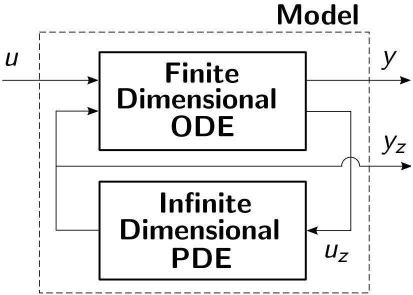


Figure: Model composed by the interconnection between ODE and PDE with **input on the ODE**.

Motivation

We focus on the **port-Hamiltonian** (PH) framework.

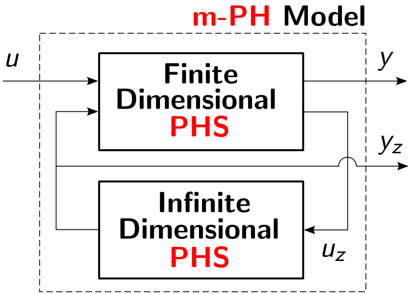


Figure: Mixed port-Hamiltonian (**m-PH**) system.

Motivation

We focus on the **port-Hamiltonian** (PH) framework.

We restrict our analysis to *models without internal dissipation*. Therefore:

- Energy is a conserved quantity
- Dissipation can be added only through the control law

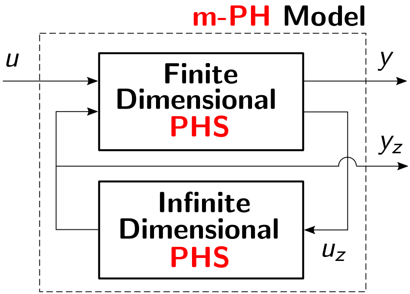


Figure: Mixed port-Hamiltonian (**m-PH**) system.

Overview

Introduction

Introduction

Let $z \in L_2([0,L],\mathbb{R}^n)$, $p \in \mathbb{R}^m$ and consider the following m-pH system

$$\begin{bmatrix} \dot{z} \\ \dot{p} \end{bmatrix} = \begin{bmatrix} P_1 \frac{\partial}{\partial \xi}\mathcal{H} + P_0 \mathcal{H} & 0 \\ -\mathcal{C}_1 \mathcal{H} & 0 \end{bmatrix} \begin{bmatrix} z \\ p \end{bmatrix} + \begin{bmatrix} 0 \\ u \end{bmatrix}$$
$$y = M^{-1} p$$

with the properties,

- $\mathcal{H} \in C^1([0,L];\mathbb{R}^{n\times n})$, $\mathcal{H}(\xi)$ is self adjoint for all $\xi \in [0,L]$ and $cI \leq \mathcal{H}(\xi) \leq CI$ for all $\xi \in [0,L]$ and some $C, c > 0$ independent of ξ
- $P_1 \in \mathbb{R}^{n\times n}$ is invertible and self adjoint, $P_0 \in \mathbb{R}^{n\times n}$ is skew adjoint
- $M \in \mathbb{R}^{m\times m}$ is $M > 0$ and symmetric

Introduction

Define the extended state $x = [z\ p]^T \in X = L_2([0,L],\mathbb{R}^n) \times \mathbb{R}^m$

$$\begin{bmatrix} \dot{z} \\ \dot{p} \end{bmatrix} = \overbrace{\begin{bmatrix} P_1 \frac{\partial}{\partial \xi}\mathcal{H} + P_0\mathcal{H} & 0 \\ -\mathcal{C}_1\mathcal{H} & 0 \end{bmatrix}}^{A_m} \begin{bmatrix} z \\ p \end{bmatrix} + \begin{bmatrix} 0 \\ u \end{bmatrix}$$

$$y = M^{-1}p$$

$$D(A_m) = \{x \in X \mid z \in H^1,\ \mathcal{B}_1(\mathcal{H}z) = M^{-1}p,\ \mathcal{B}_2(\mathcal{H}z) = 0\}.$$

Input output operators, with $u_z = \mathcal{B}_1(\mathcal{H}z)$ and $y_z = \mathcal{C}_1(\mathcal{H}z)$

$$\begin{aligned} \mathcal{B}(\mathcal{H}z) &= \begin{bmatrix} \mathcal{B}_1(\mathcal{H}z) \\ \mathcal{B}_2(\mathcal{H}z) \end{bmatrix} = \begin{bmatrix} W_{B,1} \\ W_{B,2} \end{bmatrix} \begin{bmatrix} \mathcal{H}(0)z(0,t) \\ \mathcal{H}(L)z(L,t) \end{bmatrix} \\ \mathcal{C}(\mathcal{H}z) &= \begin{bmatrix} \mathcal{C}_1(\mathcal{H}z) \\ \mathcal{C}_2(\mathcal{H}z) \end{bmatrix} = \begin{bmatrix} W_{C,1} \\ W_{C,2} \end{bmatrix} \begin{bmatrix} \mathcal{H}(0)z(0,t) \\ \mathcal{H}(L)z(L,t) \end{bmatrix} \end{aligned} \quad (1)$$

Control Design & Stability Analysis

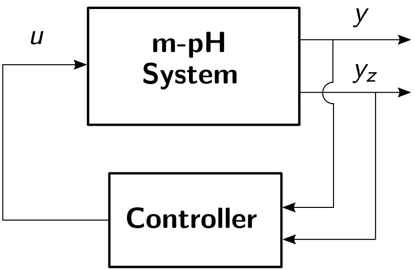


Figure: Control problem of a class of m-PH system.

Control Design & Stability Analysis

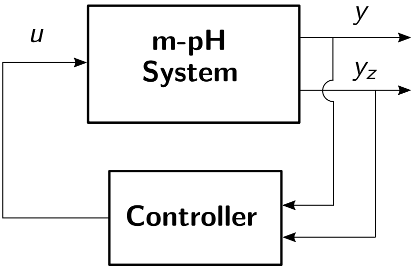


Figure: Control problem of a class of m-PH system.

Control objectives using y and y_z:

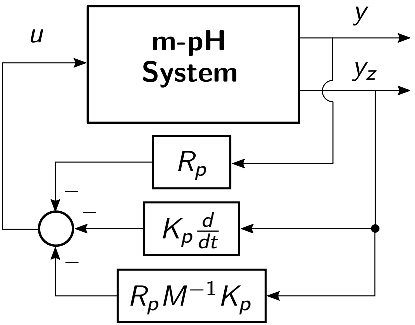
1. Stabilise the system to the origin, *i.e.* the state $x = [z\ p]^T$ has to be such that $x(t) \underset{t\to\infty}{\to} 0$
2. Stabilise the system to a desired position, *i.e.* the state $x = [z\ p\ q]^T$ has to be such that $x(t) \underset{t\to\infty}{\to} 0$

Strong dissipation feedback

Strong dissipation feedback: Exponential stability

Consider $R_p, K_p \in \mathbb{R}^{m\times m}$ diagonal and positive definite

$$u = \overbrace{-R_p M^{-1} p}^{\text{Classical dissipation}} \; \overbrace{-K_p \frac{d}{dt}(\mathcal{C}_1(\mathcal{H}z))}^{\text{Strong dissipation}} \overbrace{-R_p M^{-1} K_p \mathcal{C}_1(\mathcal{H}z)}^{\text{Additional term}} \qquad (2)$$



Strong dissipation feedback: Exponential stability

Closed-loop operator

$$\begin{bmatrix} \dot{z} \\ \dot{p} \end{bmatrix} = \overbrace{\begin{bmatrix} P_1 \frac{\partial}{\partial \xi}\mathcal{H} + P_0\mathcal{H} & 0 \\ -(I + R_p M^{-1} K_p)\mathcal{C}_1\mathcal{H} - K_p \frac{d}{dt}\mathcal{C}_1\mathcal{H} & -R_p M^{-1} \end{bmatrix}}^{A_1} \begin{bmatrix} z \\ p \end{bmatrix}$$

$$D(A_1) = \{x \in X \mid z \in H^1,\ \mathcal{B}_1(\mathcal{H}z) = M^{-1}p,\ \mathcal{B}_2(\mathcal{H}z) = 0\}$$

Strong dissipation feedback: Exponential stability

Closed-loop operator

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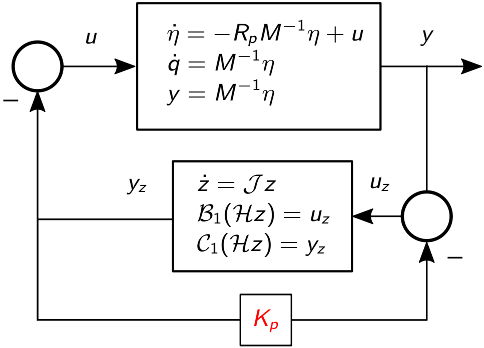
$$D(A_1) = \{x\in X \mid z\in H^1,\ \mathcal{B}_1(\mathcal{H}z) = M^{-1}p,\ \mathcal{B}_2(\mathcal{H}z) = 0\}$$

Change of variables $\boldsymbol{\eta = p + K_p\mathcal{C}_1(\mathcal{H}z)}$

$$\begin{bmatrix}\dot z\\ \dot \eta\end{bmatrix} = \overbrace{\begin{bmatrix} P_1\frac{\partial}{\partial\xi}\mathcal{H} + P_0\mathcal{H} & 0 \\ -\mathcal{C}_1\mathcal{H} & -R_pM^{-1}\end{bmatrix}}^{A}\begin{bmatrix}z\\ \eta\end{bmatrix}$$

$$D(A) = \{x\in X \mid z\in H^1,\ \mathcal{B}_1(\mathcal{H}z) = M^{-1}(\eta - K_p\mathcal{C}_1(\mathcal{H}z)),\ \mathcal{B}_2(\mathcal{H}z) = 0\}$$

Strong dissipation feedback: Exponential stability



1. Classical finite and infinite dimensional port-Hamiltonian systems
2. Boundary dissipation on the infinite dimensional system

Strong dissipation feedback: Exponential stability

Theorem 1

The operator A generates a contraction C_0-semigroup in the space $X = L_2([0,L],\mathbb{R}^n) \times \mathbb{R}^m$ equipped with the inner product

$$\langle x_1, x_2 \rangle = \langle z, \mathcal{H} z \rangle_{L_2} + \eta^T M^{-1} \eta. \tag{3}$$

Theorem 2

Assume that the input $u_z = \mathcal{B}_1(\mathcal{H}z)$ and output $y_z = \mathcal{C}_1(\mathcal{H}z)$ are such that

$$||\mathcal{H}z(0,t)||^2 \leq ||u_z(t)||^2 + ||y_z(t)||^2$$

or

$$||\mathcal{H}z(L,t)||^2 \leq ||u_z(t)||^2 + ||y_z(t)||^2$$

then the state trajectory $x(t)$ generated by the closed-loop operator A is such that $||x(t)|| \leq M_{w_0} e^{-w_0 t}$, where $M_{w_0}, w_0 > 0$.

Strong dissipation feedback: Exponential stability

Sketch of the proof.
We consider the Lyapunov function $V = \frac{1}{2}\langle z, \mathcal{H}z\rangle + \frac{1}{2}\eta^T M^{-1}\eta$,

$$\dot{V} = \langle x, Ax\rangle = -y_z^T K_p y_z - (M^{-1}\eta)^T R_p (M^{-1}\eta). \tag{4}$$

It is possible to find a $c(t) > 0$ for all $t > 0$, such that

$$V(t) \leq \frac{1}{1 + c(t)} V(0). \tag{5}$$

Since the Lyapunov function is equivalent to the state norm, we get

$$||x(t)||^2 \leq \frac{1}{1 + c(t)} ||x(0)||^2. \tag{6}$$

Hence, the semigroup $T(t)$ ($x(t) = T(t)x_0$) is such that $||T(t)|| < 1$, and therefore there exist $M_{w_0}, w_0 > 0$ such that $||x(t)|| \leq M_{w_0} e^{-w_0 t}$.

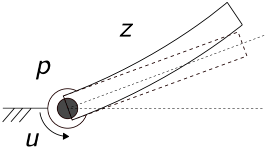
Strong dissipation & position control

Strong dissipation & position control: Asymptotic stability

Example: flexible beam clamped on a rotating inertia

port-Hamiltonian representation

$$\begin{bmatrix} \dot{z} \\ \dot{p} \end{bmatrix} = \begin{bmatrix} P_1 \frac{\partial}{\partial \xi}\mathcal{H} + P_0 \mathcal{H} & 0 \\ -\mathcal{C}_1 \mathcal{H} & 0 \end{bmatrix} \begin{bmatrix} z \\ p \end{bmatrix} + \begin{bmatrix} 0 \\ u \end{bmatrix}$$



Strong dissipation & position control: Asymptotic stability

Example: flexible beam clamped on a rotating inertia

port-Hamiltonian representation

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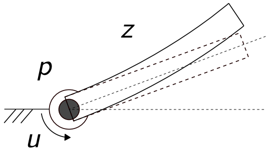


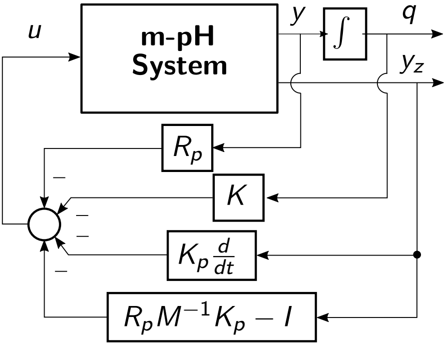
Consider the **new variable** q

1. Mechanical interpretation: displacement of the finite dimensional pH system
2. Dynamic defined as $\dot{q} = M^{-1}p$

Strong dissipation & position control: Asymptotic stability

Consider $K, R_p, K_p \in \mathbb{R}^{m\times m}$ diagonal and positive definite

$$u = -R_p M^{-1} p - Kq + (I - R_p M^{-1} K_p)\mathcal{C}_1(\mathcal{H}z) - K_p \frac{d}{dt}(\mathcal{C}_1(\mathcal{H}z))$$



Strong dissipation & position control: Asymptotic stability

Closed-loop operator, $x = [z\ p\ q]^T \in X = L_2([0,L],\mathbb{R}^n) \times \mathbb{R}^{2m}$

$$\begin{bmatrix} \dot z \\ \dot p \\ \dot q \end{bmatrix} = \overbrace{\begin{bmatrix} P_1 \frac{\partial}{\partial \xi}\mathcal{H} + P_0\mathcal{H} & 0 & 0 \\ -R_p M^{-1} K_p \mathcal{C}_1 \mathcal{H} - K_p \frac{d}{dt}\mathcal{C}_1\mathcal{H} & -R_p M^{-1} & -K \\ 0 & M^{-1} & 0 \end{bmatrix}}^{A_1} \begin{bmatrix} z \\ p \\ q \end{bmatrix}$$

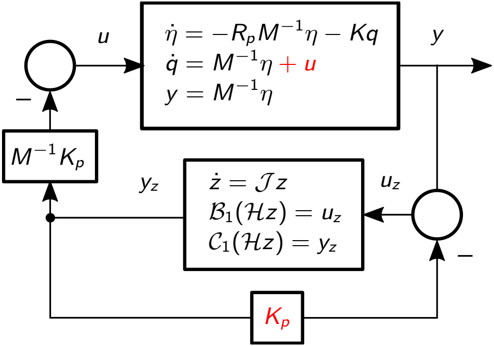
$$D(A_1) = \{x \in X \mid z \in H^1,\ \mathcal{B}_1(\mathcal{H}z) = M^{-1}p,\ \mathcal{B}_2(\mathcal{H}z) = 0\}$$

Change of variables $\boldsymbol{\eta = p + K_p \mathcal{C}_1(\mathcal{H}z)}$, $x = [z\ \eta\ q]^T$

$$\begin{bmatrix} \dot z \\ \dot \eta \\ \dot q \end{bmatrix} = \overbrace{\begin{bmatrix} P_1 \frac{\partial}{\partial \xi}\mathcal{H} + P_0\mathcal{H} & 0 & 0 \\ 0 & -R_p M^{-1} & -K \\ -M^{-1} K_p \mathcal{C}_1 \mathcal{H} & M^{-1} & 0 \end{bmatrix}}^{A} \begin{bmatrix} z \\ \eta \\ q \end{bmatrix}$$

$$D(A) = \{x \in X \mid z \in H^1,\ \mathcal{B}_1(\mathcal{H}z) = M^{-1}(\eta - K_p \mathcal{C}_1(\mathcal{H}z)),\ \mathcal{B}_2(\mathcal{H}z) = 0\}$$

Strong dissipation & position control: Asymptotic stability



1. Not classical finite dimensional port-Hamiltonian system
2. Boundary dissipation on the infinite dimensional system

Strong dissipation & position control: Asymptotic stability

Theorem 3

Assume that the control parameters are selected such that $r_i^2 \geq 2m_i k_i \; i \in \{1,\dots,m\}$, then the operator A generates a contraction C_0-semigroup in the space X equipped with the weighted norm

$$||x||_\Gamma^2 = \langle x, x\rangle_\Gamma = \langle z, \mathcal{H}z\rangle_{L_2} + \begin{bmatrix} \eta & q \end{bmatrix} \begin{bmatrix} K^{-1}M^{-1}R_pK_p^{-1} & K_p^{-1} \\ K_p^{-1} & 2KMR_p^{-1}K_p^{-1} \end{bmatrix} \begin{bmatrix} \eta \\ q \end{bmatrix}$$

Moreover A has a compact resolvent.

$\Rightarrow$ Pre-compactness of the solution set $\{x(t, x_0) \mid t \geq 0\}$

Strong dissipation & position control: Asymptotic stability

Theorem 4

Assume that the distributed parameter part of the system is **approximately observable** w.r.t. the output y_z and $\boldsymbol{z_{eq}} = \mathbf{0}$ is its only **equilibrium point**. If the control gains $k_{p,i}, r_i, k_i$ with $i = \{1, .., m\}$ are chosen such that $r_i^2 > 2m_ik_i, \quad k_{p,i} > 0$, then the origin $x_{eq} = 0$ is a globally asymptotically stable equilibrium.

Strong dissipation & position control: Asymptotic stability

Theorem 4

Assume that the distributed parameter part of the system is **approximately observable** w.r.t. the output y_z and $\boldsymbol{z_{eq} = 0}$ is its only **equilibrium point**. If the control gains $k_{p,i}, r_i, k_i$ with $i = \{1, .., m\}$ are chosen such that $r_i^2 > 2m_i k_i, \quad k_{p,i} > 0$, then the origin $x_{eq} = 0$ is a globally asymptotically stable equilibrium.

Sketch of the Proof.

Define the candidate Lyapunov function $V(x) = \frac{1}{2}\langle x, x\rangle_\Gamma$

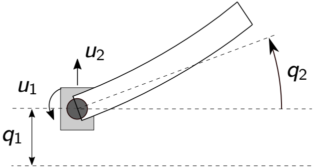
$$\dot{V} = dVAx = \langle x, Ax\rangle_\Gamma.$$

Using approximate observability and $z_{eq} = 0$, the largest invariant subset of $\mathbf{S}_0 = \{x \in X \mid \dot{V}(x) = 0\}$ corresponds to $\mathbf{S} = \{0\}$. With the LaSalle's invariance principle, $\lim_{t\to\infty} ||x(t)|| = 0$.

Comparison of the control laws

Comparison of the control laws

Example: Flexible beam clamped on a translating rotating inertia

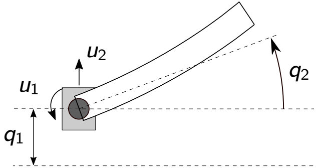


Control objectives:

1. Stabilise the system starting from any initial condition.

Comparison of the control laws

Example: Flexible beam clamped on a translating rotating inertia



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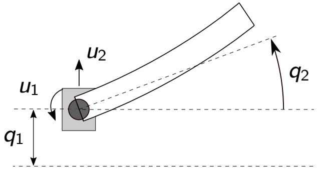
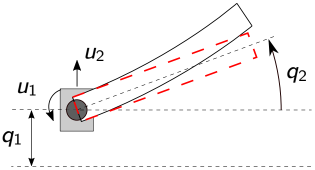


Control objectives:

1. Stabilise the flexible part of the mechanism starting from an arbitrary initial condition.
2. Stabilise the flexible part of the mechanism and position the overall mechanism in a desired configuration.

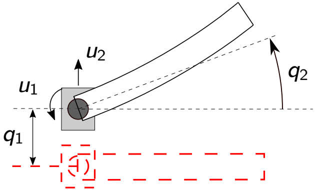
Comparison of the control laws

Strong dissipation



1. Exponential stabilisation of the flexible part of the mechanism
2. We do not have any control on the Mechanism's position

Strong dissipation & position control



1. Stabilisation of the mechanism in the desired configuration
2. We do not know the convergence rate

Conclusions, ConFlex fellowship & future plans

Conclusions & future research

Conclusions

1. Strong dissipation control: lack of direct control on the PDE's boundaries, Exponential stability, lack of position control.
2. Strong dissipation & position control: lack of direct control on the PDE's boundaries, Asymptotic stability, position control.

Future research

Modify the "Strong dissipation & position control law" such to obtain exponential stability.

Publications

Journal papers

- A. Mattioni, Y. Wu, Y. Le Gorrec. "Infinite dimensional model of a double flexible-link manipulator: The Port-Hamiltonian approach". Applied Mathematical Modelling, Elsevier. (2020)
- A. Mattioni, Y. Wu, H. Ramirez, Y. Le Gorrec, A. Macchelli. "Modelling and control of an IPMC actuated flexible structure: A lumped port Hamiltonian approach". Control Engineering Practice, Elsevier. (2020)
- A. Mattioni, Y. Wu, Y. Le Gorrec, H. Zwart. "Stabilization of a Class of Mixed ODE-PDE port-Hamiltonian Systems with Strong Dissipation Feedback". Major revision in Automatica.
- A. Mattioni, Y. Wu, Y. Le Gorrec. "Asymptotic stability of a flexible beam clamped on a rotating inertia entering in contact with the external environment". In preparation

Conference papers

- A. Mattioni, J. Toledo, Y. Le Gorrec. "Observer based nonlinear control of a rotating flexible beam". IFAC 2020 World Congress (2020).
- A. Mattioni, Y. Wu, Y. Le Gorrec, H. Zwart. "Stabilisation of a rotating beam clamped on a moving inertia with strong dissipation feedback". Control and Decision Conference (2020).
- A.Mattioni, Y. Wu, Y. Le Gorrec. "Modelling, Control and Stability Analysis of Flexible Rotating Beam's Impacts During Contact Scenario". American Control Conference (2021).
- A.Mattioni, Y. Wu, Y. Le Gorrec. "Exponential stabilization of a clamped Timoshenko beam with actuation on a tip mass". Control and Decision Conference (2021).

Secondments, Trainings & Outreach activities

Secondments

- Thales Alenia Space, Cannes, France. March - April 2019
- University of Wuppertal, Wuppertal, Germany. June 2019
- University of Twente, Enschede, The Netherlands. October - November 2019

Training and Outreach activities

- "Spring School on Theory and Applications of Port-Hamiltonian Systems", Munich, Germany. April 2019.
- 21st IFAC World Congress, Virtual. July 2020.
- 59st Control and Decision Conference, December 2020.
- 2021 American Control Conference, May 2021.

PhD & Future plans

PhD status

I got the PhD degree on "Automatic Control" by the University of Bourgogne Franche Comté on the 23$^{\text{rd}}$ April 2021.

Future plans

Post-doc position at the Gipsa-lab of the University of Grenoble on the subject: "Reinforcement Learning control of nonlinear PDE". In collaboration with Christophe Prieur and Paolo Frasca.

Thanks for your attention!