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ConFlex

Minimal Nonlinear Modal Aeroelastic Descriptions for Highly Flexible Aircraft Control

4th ConFlex Meeting

5th August, Bordeaux, France

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ConFlex
www.conflex.org

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Research motivation

Boeing's Odysseus (https://www.aurora.aero/odysseus-high-altitude-pseudo-satellite-haps/)

What are the main challenges? Proposed solutions

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- Fast/real-time control and estimation in the NL regime

Proposed solutions

- Geom. nonlinear models
- Low-order NL struct + ROM L aero
- MPC/MHE + mult. shooting + analytical sensitivities

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NASA's Helios (https://www.nasa.gov/centers/dryden/news/ResearchUpdate/Helios/)

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Realistic Aeroelasticity Sim. Host: SHARPy[1]

Structure

Displacement-based GEBT[2]

$$\mathcal{M}(\boldsymbol{\eta}) \begin{Bmatrix} \ddot{\boldsymbol{\eta}} \\ \dot{\boldsymbol{\beta}} \end{Bmatrix} + \begin{Bmatrix} \boldsymbol{Q}^S_{\mathrm{gyr}}(\boldsymbol{\eta}, \dot{\boldsymbol{\eta}}, \boldsymbol{\beta}) \\ \boldsymbol{Q}^R_{\mathrm{gyr}}(\boldsymbol{\eta}, \dot{\boldsymbol{\eta}}, \boldsymbol{\beta}) \end{Bmatrix} + \begin{Bmatrix} \boldsymbol{Q}^S_{\mathrm{stif}}(\boldsymbol{\eta}) \\ 0 \end{Bmatrix} = \begin{Bmatrix} \boldsymbol{Q}^S_{\mathrm{ext}}(\boldsymbol{\eta}, \dot{\boldsymbol{\eta}}, \boldsymbol{\beta}, \boldsymbol{\chi}) \\ \boldsymbol{Q}^R_{\mathrm{ext}}(\boldsymbol{\eta}, \dot{\boldsymbol{\eta}}, \boldsymbol{\beta}, \boldsymbol{\chi}) \end{Bmatrix}$$

Aerodynamics

[1] Del Carre A. *et al.*, "SHARPy: A dynamic aeroelastic simulation toolbox for very flexible aircraft & wind turbines", JOSS

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- $\mathcal{M}$ is the mass matrix
- $\boldsymbol{Q}$ are the discrete forces
- $\boldsymbol{\eta}_i \in \mathbb{R}^6$ are the flexible DOFs expressed in a body-attached frame
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[2] del Carre, A. *et. al.*, "Nonlinear Response of a Very Flexible Aircraft Under Lateral Gust", IFASD 2019

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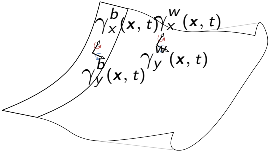

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- Nonlinear discrete-time system
- Non-penetration BCs $\rightarrow \Gamma$
- Steady forces: Joukowsky thm.
- Unsteady forces: Bernoulli

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Internal aeroelastic model for control

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Intrinsic nonlinear beam equations[1]

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Aerodynamics

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- Nonlinear? Quadratic

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- Compute LNMs $\phi_{1j}(s), \phi_{2j}(s)$
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[4] Goizueta, N. *et al.*, "Parametric Krylov-Based Model Reduction for Aeroelastic Systems," 2021 SciTech Forum

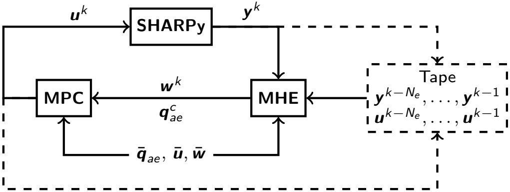
Estimation/control strategies

NMHE | NMPC

Nonlinear dynamics

Estimation/control by repeatedly solving open-loop optimal control problems s.t. dynamics & input/states constraints

Multiple shooting, SQP, analytical sensitivities



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Numerical examples I

Pazy wing, a very flexible clamped wing test case.

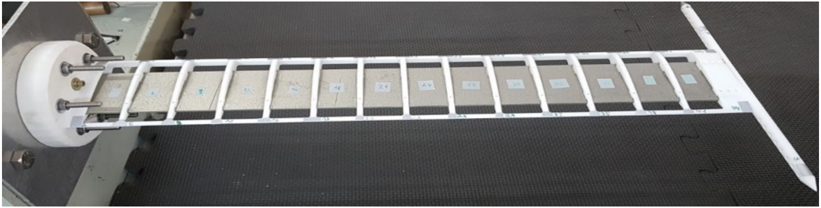

Image and data extracted from "Moving Forward with the Aeroelastic Prediction Workshop 3" presentation from NASA's Large Deflection Group

Geometry

- Chord: 100 mm
- Span: 550 mm
- Airfoil: NACA0018
- Wing-tip loading beam

Materials

- Main spar: Aluminium 7075
- Clamp base: Nylon, PA12
- Cover: Foil (Oralight)

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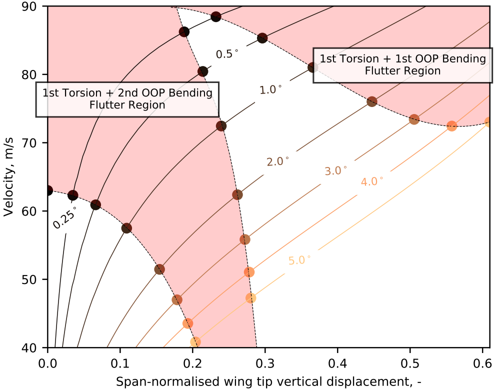
- A compelling flutter suppr. case: 2 different control mechanisms



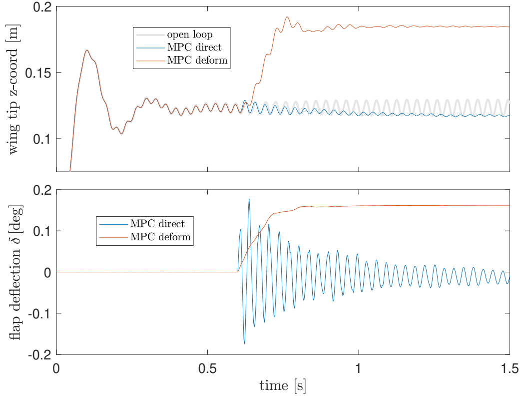
Stability analysis on deformed equilibrium point

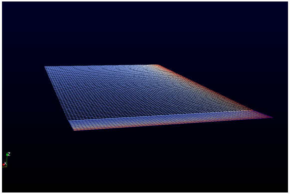
Extracted from Goizueta, N., Wynn, A., Palacios, R., Drachinksy, A., and Raveh, D. E., "Flutter predictions for very flexible wing wind tunnel test," 2021 AIAA SciTech Forum

Numerical examples I

Pazy wing, a very flexible clamped wing test case.

- A compelling flutter suppr. case: **direct actuation**

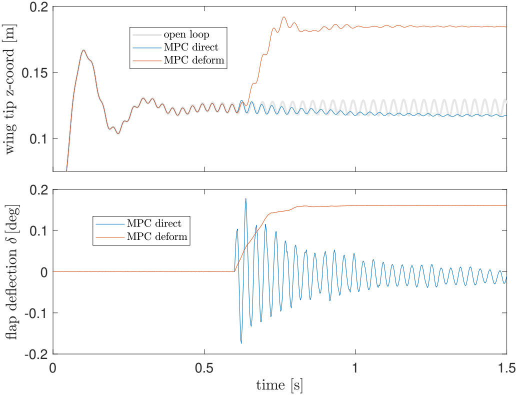


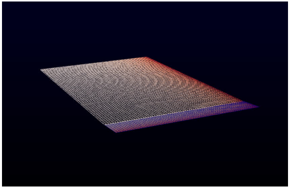


Numerical examples I

Pazy wing, a very flexible clamped wing test case.

- A compelling flutter suppr. case: **nonlinear stability leverage**





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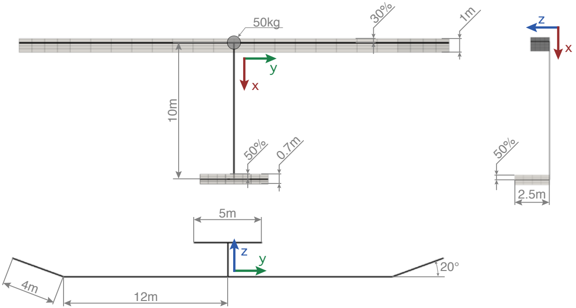
Numerical examples II

High-Altitude Long-Endurance concept very flexible aircraft



- Weight: 78.25 kg, 50 kg payload
- $\Lambda = 32$
- Flight velocity: $10 - 15\,\mathrm{m/s}$
- 1 Propeller
- T-tail tailplane: rudder and all-moving elevator

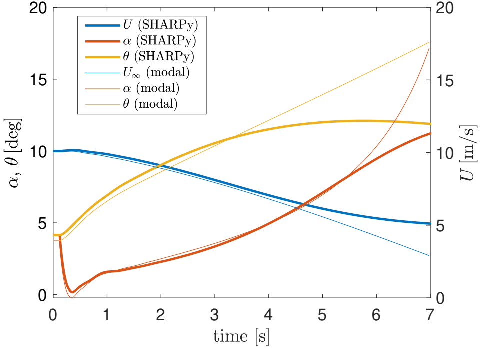
Image and data extracted from Deskos, G., del Carre, A., and Palacios, R., "Assessment of Low-Altitude Atmospheric Turbulence Models for Aircraft Aeroelasticity," Journal of Fluids and Structures, 2020.

Imperial College London

Numerical examples II

High-Altitude Long-Endurance concept very flexible aircraft

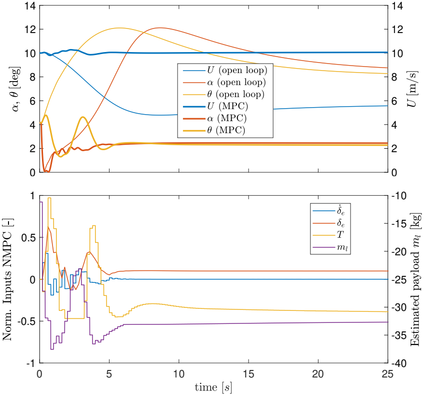
- Stabilisation after 50% payload drop. Open-loop results

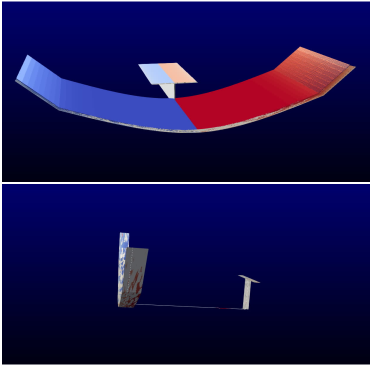


Numerical examples II

High-Altitude Long-Endurance concept very flexible aircraft

- Stabilisation after 50% payload drop. Closed-loop results





Numerical examples III

Data-driven improvements on the internal models: $y = \boldsymbol{\beta}^\top \Theta(\boldsymbol{x})$

- Gravitational model
- Aerodynamic model

Numerical examples III

Data-driven improvements on the internal models: $y = \boldsymbol{\beta}^\top \Theta(\boldsymbol{x})$

- Gravitational model $\rightarrow$ Reduce size (avoids comp. of rotations)
- Aerodynamic model $\rightarrow$ Improve accuracy (drag, dynamic pressure)

Numerical examples III

Data-driven improvements on the internal models: $y = \boldsymbol{\beta}^\top \Theta(\boldsymbol{x})$

- Gravitational model
- Aerodynamic model

Lasso regression: $X = \left(\boldsymbol{x}^1, \cdots, \boldsymbol{x}^{n_d}\right) \in \mathbb{R}^{n_x \times n_d}$, $Y = \left(y^1, \cdots, y^{n_d}\right) \in \mathbb{R}^{1 \times n_d}$

- Quadratic models
- Physics-constrained

$$\min_{\boldsymbol{\beta}} \|Y - \boldsymbol{\beta}^\top \Theta(X)\|_2^2 + \lambda \|\boldsymbol{\beta}\|_1, \quad \Theta(X) = \begin{pmatrix} 1 & & 1 \\ x_1^1 & & x_1^{n_d} \\ \vdots & & \vdots \\ x_{n_x}^1 & & x_{n_x}^{n_d} \\ (x_1^1)^2 & \cdots & (x_1^{n_d})^2 \\ x_1^1 x_2^1 & & x_1^{n_d} x_2^{n_d} \\ \vdots & & \vdots \\ (x_{n_x}^1)^2 & & (x_{n_x}^{n_d})^2 \end{pmatrix}$$

Numerical examples III

Data-driven improvements on the internal models: $y = \boldsymbol{\beta}^\top \Theta(\boldsymbol{x})$

- Gravitational model
- Aerodynamic model

Lasso regression: $X = \left(\boldsymbol{x}^1, \cdots, \boldsymbol{x}^{n_d}\right) \in \mathbb{R}^{n_x \times n_d}$, $Y = \left(y^1, \cdots, y^{n_d}\right) \in \mathbb{R}^{1 \times n_d}$

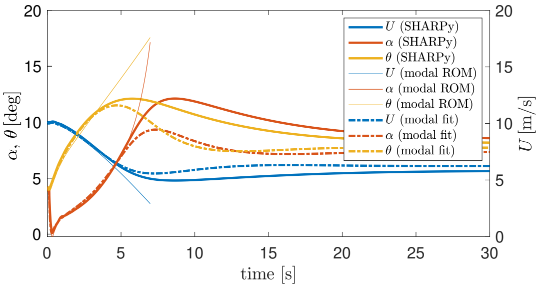
- Quadratic models $\rightarrow$ Consistency, analytical manipulation
- Physics-constrained $\rightarrow$ Trim, stability [Schlegel & Noack, 2015]

$$\min_{\boldsymbol{\beta}} \|Y - \boldsymbol{\beta}^\top \Theta(X)\|_2^2 + \lambda \|\boldsymbol{\beta}\|_1, \quad \Theta(X) = \begin{pmatrix} 1 & & 1 \\ x_1^1 & & x_1^{n_d} \\ \vdots & & \vdots \\ x_{n_x}^1 & & x_{n_x}^{n_d} \\ (x_1^1)^2 & \cdots & (x_1^{n_d})^2 \\ x_1^1 x_2^1 & & x_1^{n_d} x_2^{n_d} \\ \vdots & & \vdots \\ (x_{n_x}^1)^2 & & (x_{n_x}^{n_d})^2 \end{pmatrix}$$

Numerical examples III

Data-driven improvements on the internal models: $y = \boldsymbol{\beta}^\top \Theta(\boldsymbol{x})$

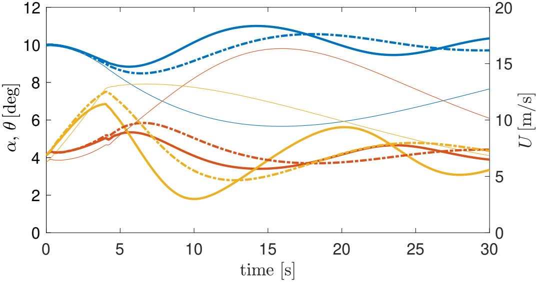
- Open-loop improvements: payload drop release
- Effect on MPC/MHE closed-loop performance: improved estimation, inconclusive effect on control



Numerical examples III

Data-driven improvements on the internal models: $y = \boldsymbol{\beta}^\top \Theta(\boldsymbol{x})$

- Open-loop improvements: elevator deflection
- Effect on MPC/MHE closed-loop performance:
  improved estimation, inconclusive effect on control



Numerical examples III

Data-driven improvements on the internal models: $y = \boldsymbol{\beta}^\top \Theta(\boldsymbol{x})$

- Open-loop improvements:
- Effect on MPC/MHE closed-loop performance: improved estimation, inconclusive effect on control

Concluding remarks

Summary of research achievements

- Nonlinear low-order internal models for control
- Application of data-driven methods in aeroelasticity modelling
- Application examples of the framework with novel nonlinear-exploiting control mechanisms
- Framework opens the door to real-time nonlinear control of very flexible aircraft.

Conflex Fellowship Summary

Publications

Journal

- M. Artola *et al.*, "Generalized Kelvin-Voigt Damping Model for Geometrically-Nonlinear Beams", AIAA Journal, Vol. 59, Num. 1, pp. 356-365.
- M. Artola *et al.*, "Aeroelastic Control and Estimation with a Minimal Nonlinear Modal Description", AIAA Journal, Vol. 59, Num. 7, pp. 2697-2713.
- M. Artola *et al.*, "Modal-Based Nonlinear Model Predictive Control for 3D Very Flexible Structures", IEEE Transactions on Automatic Control, Vol. 67, Num. 5.

Conference

- M. Artola *et al.*, "A Nonlinear Modal-Based Framework for Low Computational Cost Optimal Control of 3D Very Flexible Structures", 2019 18th European Control Conference (ECC), Naples, Italy, Jun 2019, pp. 3836-3841.
- M. Artola *et al.*, "Modal-Based Nonlinear Estimation and Control for Highly Flexible Aeroelastic Systems", AIAA Scitech 2020 Forum, Orlando, FL, Jan 2020.
- M. Artola *et al.*,"Modal-Based Model Predictive Control of Multibody Very Flexible Structures", 21st IFAC World Congress 2020, Berlin, Germany, Jul 2020.
- M. Artola *et al.*, "Proof of Concept for a Hardware-in-the-Loop Nonlinear Control Framework for Very Flexible Aircraft", AIAA Scitech 2021 Forum, Nashville, TN, Jan 2021.

Accepted (**in colab. with ESR 5 Charlotte Rodriguez!**)

- M. Artola, C. Rodriguez *et al.* "Optimisation of Region of Attraction Estimates for the Exponential Stabilisation of the Intrinsic Geometrically Exact Beam Model", 2021 IEEE Conference on Decision and Control (CDC).

Conflex Fellowship Summary

PhD status

Thesis submitted only yesterday. Awaiting for viva date.

Future employment prospects

Interested in R&D Engineering positions in the industry

Minimal Nonlinear Modal Aeroelastic Descriptions for Highly Flexible Aircraft Control

4th ConFlex Meeting

5th August, Bordeaux, France

M. Artola*, Dr. A. Wynn, Prof. R. Palacios

Department of Aeronautics, Imperial College London

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www.conflex.org

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