Abstract nonlinear control systems

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This project has received funding from the European Union's Horizon 2020 research and innovation programme under the Marie Sklodowska-Curie grant agreement No 765579.

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Overview

Initial Value Problem (Existence and uniqueness)

$$\dot{x}(t) = f(x,t), \qquad x(t_0) = x_0, \tag{1}$$

where $x(t) \in X \subseteq \mathbb{R}^n$.

Initial Value Problem (Existence and uniqueness)

$$\dot{x}(t) = f(x,t), \qquad x(t_0) = x_0, \tag{1}$$

where $x(t) \in X \subseteq \mathbb{R}^n$. Let $f(x,t)$ be piecewise continuous in t and locally Lipschitz in x i.e. for each $x_0 \in X \subseteq \mathbb{R}^n$, there is a real number $r > 0$ such that the ball $\mathcal{B}_r(x_0)$ is contained in X and $\exists$ an L such that

Locally Lipschitz continuous

$$\|f(x,t) - f(y,t)\| \leqslant L\|x(t) - y(t)\| \qquad \forall\, x(t), y(t) \in \mathcal{B}_r(x_0), \qquad \forall\, t \in [t_0, t_1].$$

Then (1) has a unique solution $x : [t_0, t_1] \to X$.

Cauchy Problem

Abstract Cauchy problem

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Proposition

If $A : \mathcal{D}(A) \subset X \to X$ is the generator of a strongly continuous semigroup $(\mathbb{T}_t)_{t\geqslant 0}$ on X, then $x(t) = \mathbb{T}_t x_0$ is continuous as a $\mathcal{D}(A)$-valued function and is the unique solution of (2).

C_0 semigroups

Mappings $\mathbb{T} : \mathbb{R}_+ \to X$ which satisfy:

Functional equation

$$\begin{cases} \mathbb{T}(t+s) = \mathbb{T}(t)\mathbb{T}(s) & \forall\, t, s \geqslant 0, \\ \mathbb{T}(0) = I. \end{cases} \tag{3}$$

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Strong continuity

$$\lim_{t\to 0, t>0} \mathbb{T}(t)x = x \qquad \forall\, x \in X. \tag{4}$$

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$$Ax = \lim_{t \to 0, t>0} \frac{1}{t}\left[\mathbb{T}x - x\right], \tag{5}$$

$$\mathcal{D}(A) = \{x \in X \mid \text{the above limit exists}\}. \tag{6}$$

For any operator $A \in \mathcal{L}(X)$, the C_0 semigroup generated is $\mathbb{T}(t) = e^{At}$.

Hille-Yosida, 1948

For a linear operator A on a Banach space X, the following properties are equivalent:

- A generates a strongly continuous contraction semigroup.
- A is closed, densely defined and for every $\lambda \in \mathbb{C}$ with $\mathrm{Re}\lambda > 0$ one has $\lambda \in \rho(A)$ and
$$\|\lambda(\lambda I - A)^{-1}\| \leqslant \frac{1}{\mathrm{Re}\lambda} \qquad (7)$$

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Feller, Miyadera, Phillips, 1952

Let A be a linear operator on a Banach space X and let $w \in \mathbb{R}$, $M \geqslant 1$ be constants. Then the following properties are equivalent:

- A generates a strongly continuous semigroup $(\mathbb{T}_t)_{t \geqslant 0}$ satisfying $\|\mathbb{T}_t\| \leqslant Me^{wt}$.
- A is closed, densely defined and for every $\lambda \in \mathbb{C}$ with $\mathrm{Re}\lambda > w$ one has $\lambda \in \rho(A)$ and

$$\|\lambda(\lambda I - A)^{-n}\| \leqslant \frac{M}{(\mathrm{Re}\lambda - w)^n} \qquad \forall\, n \in \mathbb{N}. \tag{8}$$

Lumer, Phillips, 1961

For a densely defined, dissipative operator A on a Banach space X the following statements are equivalent:

- The operator A generates a contraction semigroup.
- $\mathrm{Ran}(\lambda I - A) = X$ for some (hence all) $\lambda > 0$.

Operator A is dissipative if for some $\lambda \in \mathbb{C}$ such that $\mathrm{Re}\lambda > 0$ we have that

$$\|(\lambda I - A)x\| \geqslant \lambda \|x\|.$$

Linear time invariant control systems (system node)

$$\dot{x}(t) = Ax(t) + Bu(t), \tag{9}$$

$$y(t) = \bar{C}x(t) + Du(t). \tag{10}$$

where $x(0) = x_0 \in \mathcal{D}(A)$ and A, B, C, D, are linear operators such that $A : \mathcal{D}(A) \to X$, $B \in \mathcal{L}(U, X_{-1})$, $C \in \mathcal{L}(X_1, Y)$ and $D \in \mathcal{L}(U, Y)$. $\bar{C}$ is the extension of C (not necessarily unique) to X.

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Well-posed solutions

existence + uniqueness + continuous dependence = well-posedness

Well-posed linear system

Well-posed linear system Σ

$$\begin{bmatrix} x(\tau) \\ \mathbf{P}_\tau y \end{bmatrix} = \underbrace{\begin{bmatrix} \mathbb{T}_\tau & \Phi_\tau \\ \Psi_\tau & \mathbb{F}_\tau \end{bmatrix}}_{\Sigma_\tau} \begin{bmatrix} x_0 \\ \mathbf{P}_\tau u \end{bmatrix}. \tag{11}$$

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Family of operators

$$x(\tau) = \mathbb{T}_\tau x_0 + \underbrace{\int_0^\tau \mathbb{T}_{\tau-\sigma} B u(\sigma) \mathrm{d}\sigma}_{\Phi_\tau u} \quad \forall x_0 \in \mathcal{D}(A), \quad \forall u \in L^2([0,\infty); U), \tag{12}$$

$$\mathbf{P}_\tau y = \underbrace{\bar{C}\mathbb{T}_t x_0}_{(\Psi_\tau x_0)(t)} + \underbrace{\bar{C} \int_0^t \mathbb{T}_{t-\sigma} B u(\sigma) \mathrm{d}\sigma + D u(t)}_{(\mathbb{F}_\tau u)(t)} \quad \forall x_0 \in \mathcal{D}(A),\ t \in [0,\tau]. \tag{13}$$

Motivating Example

Wind tower coupled with TMD

- Tower (described by homogeneous Euler-Bernoulli beam model) is clamped at bottom.
- Nacelle considered as a rigid body has mass M and is mounted on top.
- Tuned mass damper (TMD) used to dampen the vibrations.

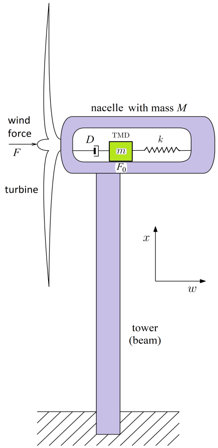

Figure: Wind tower coupled with tuned mass damper (TMD).

Wind turbine tower coupled with TMD, defined for $(x,t) \in ((0,l) \times [0,\infty))$ is

PDEs representing Euler-Bernoulli beam coupled with TMD

$$\begin{cases} \rho w_{tt}(x,t) + EI w_{xxxx}(x,t) = 0, \\ w(0,t) = 0, \qquad w_x(0,t) = 0, \\ M w_{tt}(l,t) - EI w_{xxx}(l,t) = F(t) - D[w_t(l,t) - \xi_t(t)] - k[w(l,t) - \xi(t)], \\ J w_{xtt}(l,t) + EI w_{xx}(l,t) = 0, \\ m\xi_{tt}(t) = D[w_t(l,t) - \xi_t(t)] + k[w(l,t) - \xi(t)], \end{cases}$$

EI is the flexural rigidity, ρ is the mass density and J is the moment of inertia.

Wind tower coupled with TMD

State space $X = \mathcal{H}_l^2(0,l) \times L^2[0,l] \times \mathbb{C}^4$, and $U = Y = \mathbb{C}$.

State space representation

$$\dot{x}(t) = Ax(t) + Bu(t),$$
$$y(t) = \bar{C}x(t).$$

where $A : \mathcal{D}(A) \to X$, $B \in \mathcal{L}(U, X_{-1})$, $C \in \mathcal{L}(X_1, Y)$.

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where $A : \mathcal{D}(A) \to X$, $B \in \mathcal{L}(U, X_{-1})$, $C \in \mathcal{L}(X_1, Y)$.

- This linear system Σ is well-posed.
- Σ strongly stable on X.

Wind tower-TMD system with nonlinear damping

Wind tower-TMD system with friction term

$$\begin{cases} \rho w_{tt}(x,t) + EIw_{xxxx}(x,t) = 0, \\ w(0,t) = 0, \qquad w_x(0,t) = 0, \\ Mw_{tt}(l,t) - EIw_{xxx}(l,t) = F(t) - D[w_t(l,t) - \xi_t(t)] - k[w(l,t) - \xi(t)] \\ \qquad\qquad - f_0 \operatorname{sign}[w_t(l,t) - \xi_t(t)], \\ Jw_{xtt}(l,t) + EIw_{xx}(l,t) = 0, \\ m\xi_{tt}(t) = D[w_t(l,t) - \xi_t(t)] + k[w(l,t) - \xi(t)] + f_0 \operatorname{sign}[w_t(l,t) - \xi_t(t)], \end{cases}$$

where

$$\operatorname{sign}(v) = \begin{cases} 1 & \text{if } v > 0, \\ -1 & \text{if } v < 0, \\ [-1,1] & \text{if } v = 0. \end{cases} \tag{14}$$

Objectives

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1. Well-posedness of the coupled wind tower with nonlinear damping term $\mathcal{N}$.

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2. Extend the study to nonlinear infinite dimensional system $\Sigma^{\mathcal{N}}$ represented by:

Nonlinear infinite dimensional system

$$\dot{x}(t) \in Ax(t) - \mathcal{N}(x(t)) + Bu(t)\,, \tag{15}$$

$$y(t) = \bar{C}x(t) + Du(t)\,. \tag{16}$$

Lax-Phillips semigroup

Lax-Phillips semigroup

Assuming that Σ is a well-posed system on Y, X, U. Let $\mathcal{U} = L^2([0,\infty); U)$ and $\mathcal{Y} = L^2((-\infty, 0]; Y)$. For each $[y_0, x_0, u_0] \in \mathcal{Y} \times X \times \mathcal{U}$ and $t \geqslant 0$ we define on $\mathcal{Y} \times X \times \mathcal{U}$ the operator $\mathfrak{T}_t$ by

Lax-Phillips model of semigroup

$$\mathfrak{T}_t \begin{bmatrix} y_0 \\ x_0 \\ u_0 \end{bmatrix} = \begin{bmatrix} \mathcal{S}_{-t} & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & \mathbf{S}_t^* \end{bmatrix} \begin{bmatrix} I & \Psi_t & \mathbb{F}_t \\ 0 & \mathbb{T}_t & \Phi_t \\ 0 & 0 & I \end{bmatrix} \begin{bmatrix} y_0 \\ x_0 \\ u_0 \end{bmatrix}. \tag{17}$$

Then $\mathfrak{T} = (\mathfrak{T}_t)_{t \geqslant 0}$ is a strongly continuous semigroup on $\mathcal{Y} \times X \times \mathcal{U}$. $\mathfrak{T}_t \left[\begin{smallmatrix} y_0 \\ x_0 \\ u_0 \end{smallmatrix}\right]$ contains all the information of system Σ.

Lax-Phillips semigroup

The following conditions are equivalent:

1. Σ is scattering passive, i.e. the following inquality holds for all $\tau \geqslant 0$:

$$\|x(\tau)\|^2 + \int_0^\tau \|y(t)\|^2 \mathrm{d}t \leqslant \|x(0)\|^2 + \int_0^\tau \|u(t)\|^2 \mathrm{d}t, \tag{18}$$

2. The Lax-Phillips semigroup induced by Σ is contractive.
3. $\|\mathfrak{T}_t\| = 1$ for all $t \geqslant 0$.

Generator of Lax-Phillips semigroup

Generator of $\mathfrak{T}$

$$\begin{bmatrix} y_0' \\ A\&B \left[\begin{smallmatrix} x_0 \\ u_0(0) \end{smallmatrix}\right] \\ u_0' \end{bmatrix} = \mathfrak{A} \begin{bmatrix} y_0 \\ x_0 \\ u_0 \end{bmatrix} = \begin{bmatrix} \left[\frac{\mathrm{d}}{\mathrm{d}\xi}\right]_{\mathcal{Y}} & \delta_0 \bar{C} & \delta_0 D \delta_0^* \\ 0 & A & B\delta_0^* \\ 0 & 0 & \left[\frac{\mathrm{d}}{\mathrm{d}\xi}\right]_{\mathcal{U}} \end{bmatrix} \begin{bmatrix} y_0 \\ x_0 \\ u_0 \end{bmatrix} \qquad \forall \begin{bmatrix} y_0 \\ x_0 \\ u_0 \end{bmatrix} \in \mathcal{D}(\mathfrak{A}). \quad (19)$$

Generator of Lax-Phillips semigroup

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Everywhere defined perturbation of $\mathfrak{T}$

$$\mathfrak{A}^{\mathcal{N}} \begin{bmatrix} y_0 \\ x_0 \\ u_0 \end{bmatrix} = \begin{bmatrix} \left[\frac{\mathrm{d}}{\mathrm{d}\xi}\right]_{\mathcal{Y}} & \delta_0 \bar{C} & \delta_0 D \delta_0^* \\ 0 & A - \mathcal{N} & B\delta_0^* \\ 0 & 0 & \left[\frac{\mathrm{d}}{\mathrm{d}\xi}\right]_{\mathcal{U}} \end{bmatrix} \begin{bmatrix} y_0 \\ x_0 \\ u_0 \end{bmatrix} \qquad \forall \begin{bmatrix} y_0 \\ x_0 \\ u_0 \end{bmatrix} \in \mathcal{D}(\mathfrak{A}). \quad (20)$$

Strongly continuous semigroup of nonlinear operators

Given a strongly continuous semigroup of nonlinear $\mathfrak{T}$ on real Hilbert space Z, the generator is defined as:

$$\mathfrak{A}^0 z = \lim_{t \to 0, t>0} \frac{1}{t} \left[\mathfrak{T}_t z - z \right], \tag{21}$$

$$\mathcal{D}(\mathfrak{A}^0) = \{ z \in Z \mid \text{the above limit exists} \}. \tag{22}$$

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Contractive semigroup of nonlinear operators

Assume that $\mathfrak{T}$ is contractive, i.e.

$$\|\mathfrak{T}_t z_1 - \mathfrak{T}_t z_2\| \leqslant \|z_1 - z_2\| \qquad \forall\, z_1, z_2 \in Z, t \geqslant 0.$$

Then $\mathfrak{A}^0$ is densely defined and dissipative.

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Then $\mathfrak{A}^0$ is densely defined and dissipative.

- $\mathfrak{A}^0$ has a maximal dissipative extension $\mathfrak{A}$ (possibly set-valued) with $\mathcal{D}(\mathfrak{A}) = \mathcal{D}(\mathfrak{A}^0)$.
- If $z_0 \in \mathcal{D}(\mathfrak{A})$ then $\mathfrak{A}^0 z_0$ is the unique element of smallest norm in $\mathfrak{A} z_0$.
- $z(t) = \mathfrak{T}_t z_0$ is Lipschitz continuous and right differentiable at every $t \geqslant 0$.

Main Results

Main result

Theorem

- Let $\Sigma = \left[\begin{smallmatrix} \mathbb{T} & \Phi \\ \Psi & \mathbb{F} \end{smallmatrix}\right]$ be a scattering passive linear system on Y, X, U, described by the operators $A : \mathcal{D}(A) \to X$, $B \in \mathcal{L}(U, X_{-1})$, $C \in \mathcal{L}(X_1, Y)$ and $D \in \mathcal{L}(U, Y)$.
- Let $\mathcal{N}$ be a (set-valued) maximal monotone operator with $\mathcal{D}(\mathcal{N}) = X$.

Then there exists a time-invariant well-posed nonlinear system $\Sigma^{\mathcal{N}}$

$$\dot{x}(t) \in \begin{bmatrix} A - \mathcal{N} & B \end{bmatrix} \begin{bmatrix} x(t) \\ u(t) \end{bmatrix}, \tag{23}$$

$$y(t) = \begin{bmatrix} \bar{C} & D \end{bmatrix} \begin{bmatrix} x(t) \\ u(t) \end{bmatrix}. \tag{24}$$

Moreover, $\Sigma^{\mathcal{N}}$ is incrementally scattering passive.

Incrementally scattering passive

Let x_{01}, $x_{02} \in X$ and u_1, $u_2 \in L^2_{\text{loc}}([0,\infty);U)$, then corresponding state trajectories x_1, x_2 and outputs y_1 ,y_2 of $\Sigma^{\mathcal{N}}$ satisfy for all $\tau \geqslant 0$,

Energy balance inequality:

$$\begin{aligned}\|x_1(\tau) - x_2(\tau)\|^2 &+ \int_0^\tau \|y_1(t) - y_2(t)\|^2 \mathrm{d}t \\ &\leqslant \|x_{01} - x_{02}\|^2 + \int_0^\tau \|u_1(t) - u_2(t)\|^2 \mathrm{d}t. \qquad (25)\end{aligned}$$

Perturbed operator $\mathfrak{A}$

Perturbted $\mathfrak{A}$

$$\mathfrak{A}^{\mathcal{N}} = \begin{bmatrix} \frac{\mathrm{d}}{\mathrm{d}\xi} & \delta_0 \bar{C} & \delta_0 D \delta_0^* \\ 0 & A - \mathcal{N} & B\delta_0^* \\ 0 & 0 & \frac{\mathrm{d}}{\mathrm{d}\xi} \end{bmatrix}, \tag{26}$$

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Splitting $\mathfrak{A}^{\mathcal{N}}$

$$\mathfrak{A}^{\mathcal{N}} = \underbrace{\begin{bmatrix} \frac{\mathrm{d}}{\mathrm{d}\xi} & \delta_0 \bar{C} & \delta_0 D \delta_0^* \\ 0 & A & B\delta_0^* \\ 0 & 0 & \frac{\mathrm{d}}{\mathrm{d}\xi} \end{bmatrix}}_{\mathfrak{A}} + \underbrace{\begin{bmatrix} 0 & 0 & 0 \\ 0 & -\mathcal{N} & 0 \\ 0 & 0 & 0 \end{bmatrix}}_{\tilde{\mathcal{N}}}. \tag{27}$$

$\mathcal{D}(\tilde{\mathcal{N}}) = \mathcal{Y} \times X \times \mathcal{U}$. Therefore, $\mathcal{D}(\mathfrak{A}) \cap \left(\mathrm{int}\mathcal{D}(\tilde{\mathcal{N}})\right) = \mathcal{D}(\mathfrak{A})$, which is dense.

Future plans and potential research directions

- Consider the case when $\mathcal{D}(\mathcal{N}) \subset X$.
- Generalized representation of nonlinear infinite dimensional systems.
- Numerical analysis of Wind turbine tower-TMD system to study the effect nonlinear damping.
- Stability analysis.

List of publications

- Shantanu Singh, Marius Tucsnak, and George Weiss (2020).
  Non-linear damping for scattering-passive systems in the Maxwell class.
  IFAC-PapersOnLine, vol. 53, pp. 7458-7465.
  https://doi.org/10.1016/j.ifacol.2020.12.1298

- Shantanu Singh, George Weiss and Marius Tucsnak (2021).
  Abstract nonlinear control systems.
  Accepted in CDC-2021.

- Shantanu Singh, George Weiss, and Marius Tucsnak (2021).
  A class of incrementally scattering-passive nonlinear systems.
  Under review in Automatica.

Thank You.