



Control and Design for Fluids and Structures

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ConFlex-UAM Workshop, February 2019



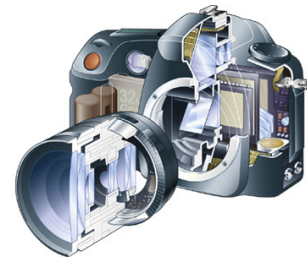
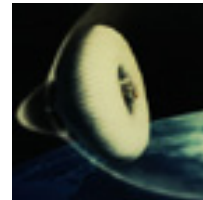
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Contemporary Applications

Control theory and applications

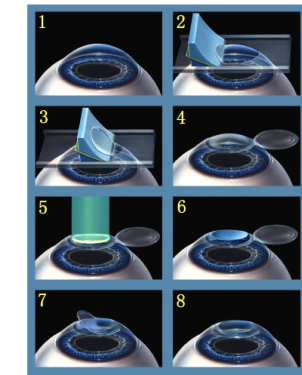
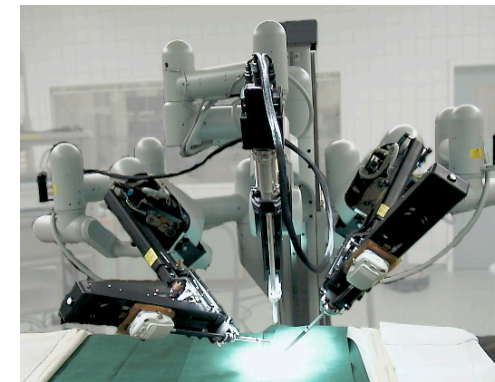
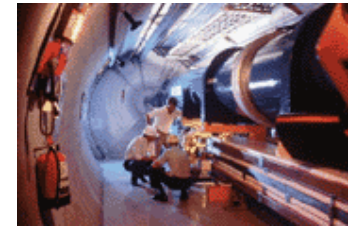
Mechanics

Vehicles (guidance, dampers, ABS, ESP, ...),
Aeronautics, aerospace (shuttle, satellites), robotics



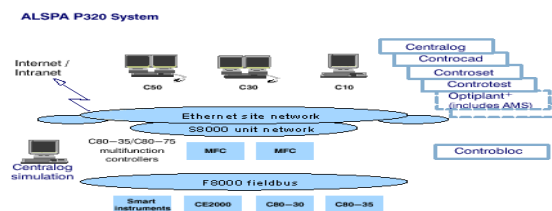
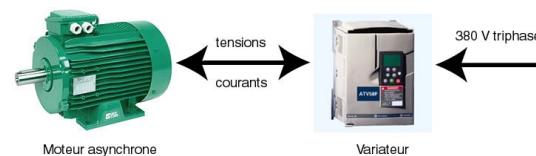
Biology, medicine

Predator-prey systems, bioreactors, epidemiology,
medicine (pacemakers, laser surgery)



Electricity, electronics

RLC circuits, thermostats, regulation, refrigeration, computers, internet
and telecommunications in general, photography and digital video



Economics

Gain optimization, control of financial flux,
Market prevision



| | | | | |
|--------|--------|--------|--------|--------|
| 19.96 | 37.52 | 17.12 | -0.06 | 1.81% |
| 38.75 | 17.02 | 42.15 | +0.75 | 0.48% |
| 17.47 | 40.86 | 27.09 | +0.13 | 2.09% |
| 42.45 | 26.07 | 22.47 | +0.46 | -5.12% |
| 27.15 | 21.71 | 23.37 | -1.26 | 3.30% |
| 22.59 | 22.74 | 391.66 | +12.51 | 0.78% |
| 29.97 | 22.74 | 391.66 | +0.74 | 1.69% |
| 391.70 | 377.43 | 25.22 | +0.42 | 1.22% |
| 391.70 | 377.43 | 25.22 | +0.30 | 1.22% |
| 391.70 | 377.43 | 25.22 | +0.30 | 1.22% |

Chemistry

Chemical kinetics, engineering process, petroleum, distillation, petrochemical industry

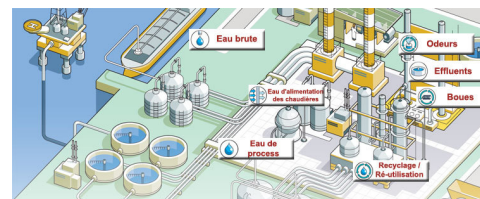
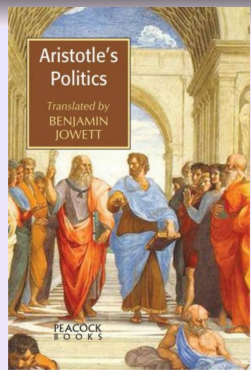


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Origins

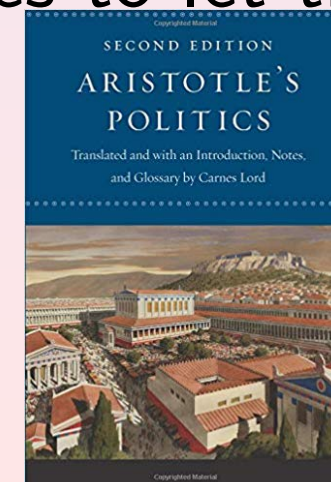
THE ORIGINS:



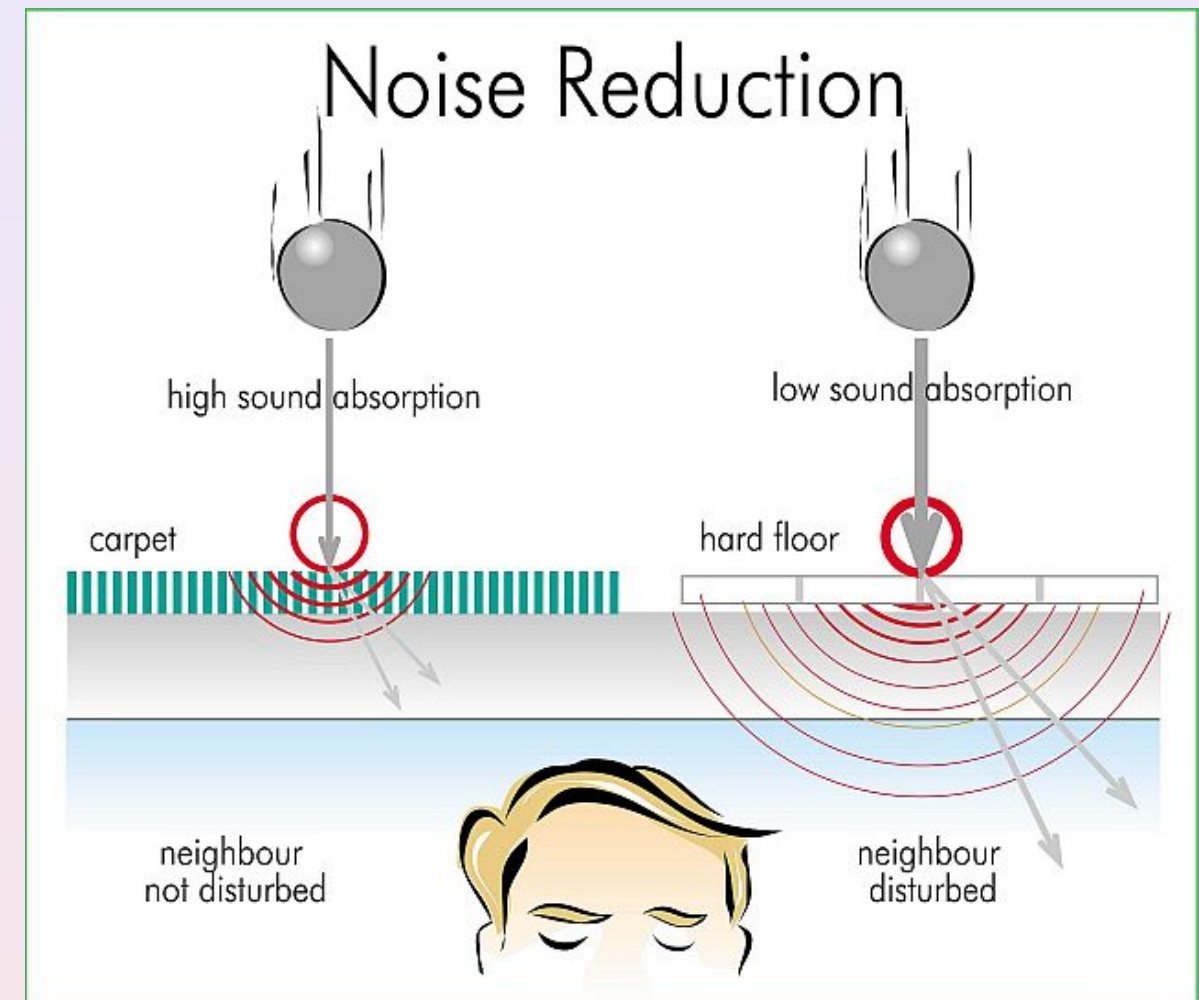
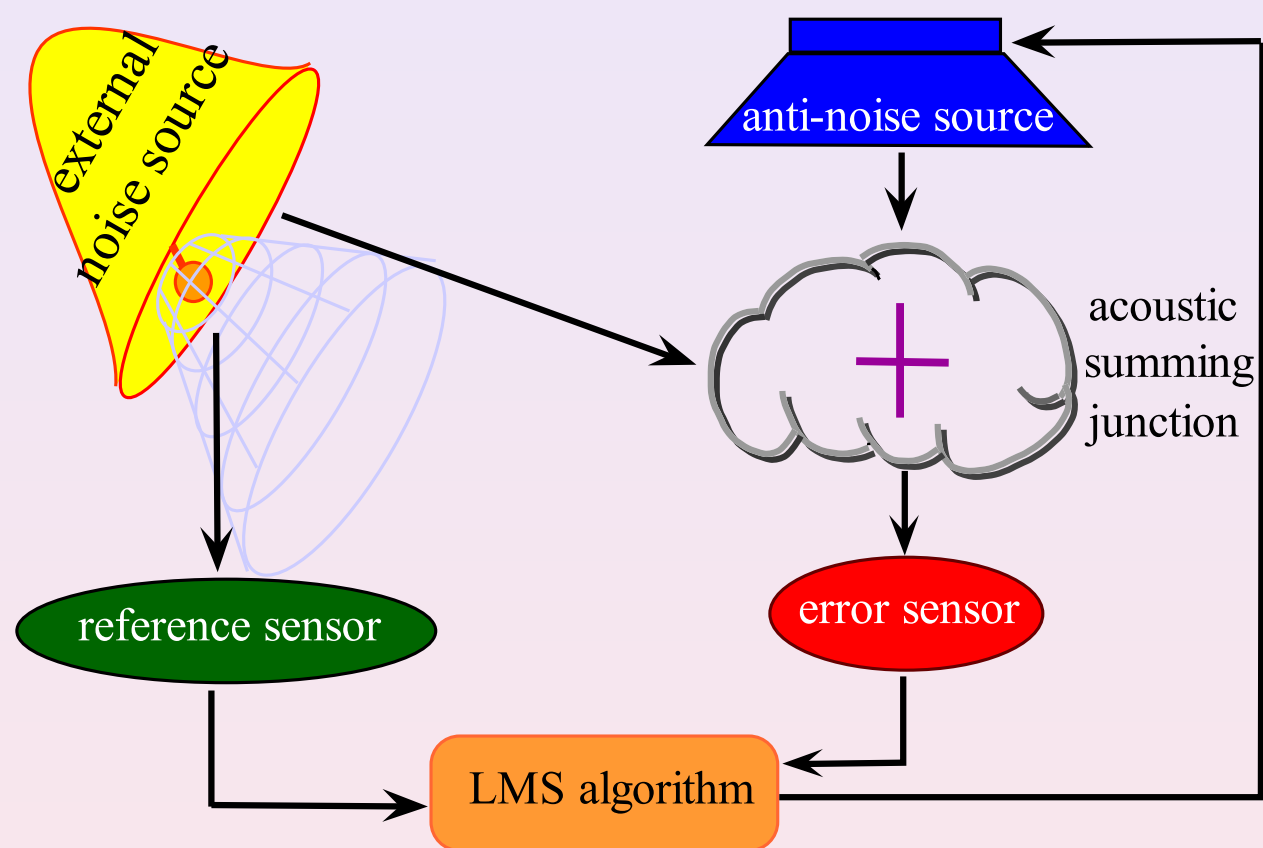
*“... if every instrument could accomplish its own work, obeying or anticipating the will of others ...
if the shuttle weaved and the pick touched the lyre without a hand to guide them, chief workmen would not need servants, nor masters slaves.”*

Chapter 3, Book 1, of the monograph “Politics” by [Aristotle](#) (384-322 B. C.).

Main motivation: The need of automatizing processes to let the human being gain in liberty, freedom, and quality of life.

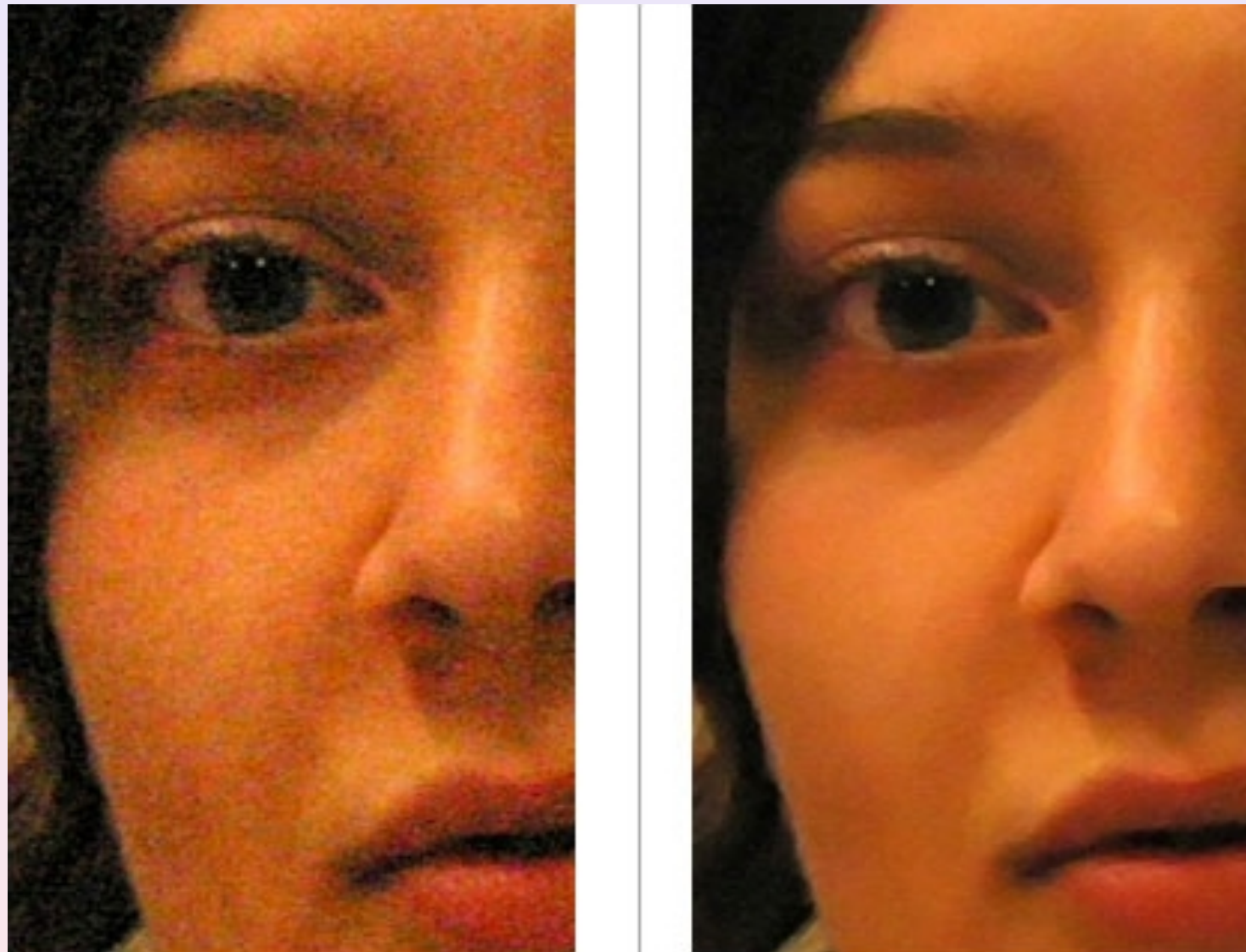


An example: noise reduction

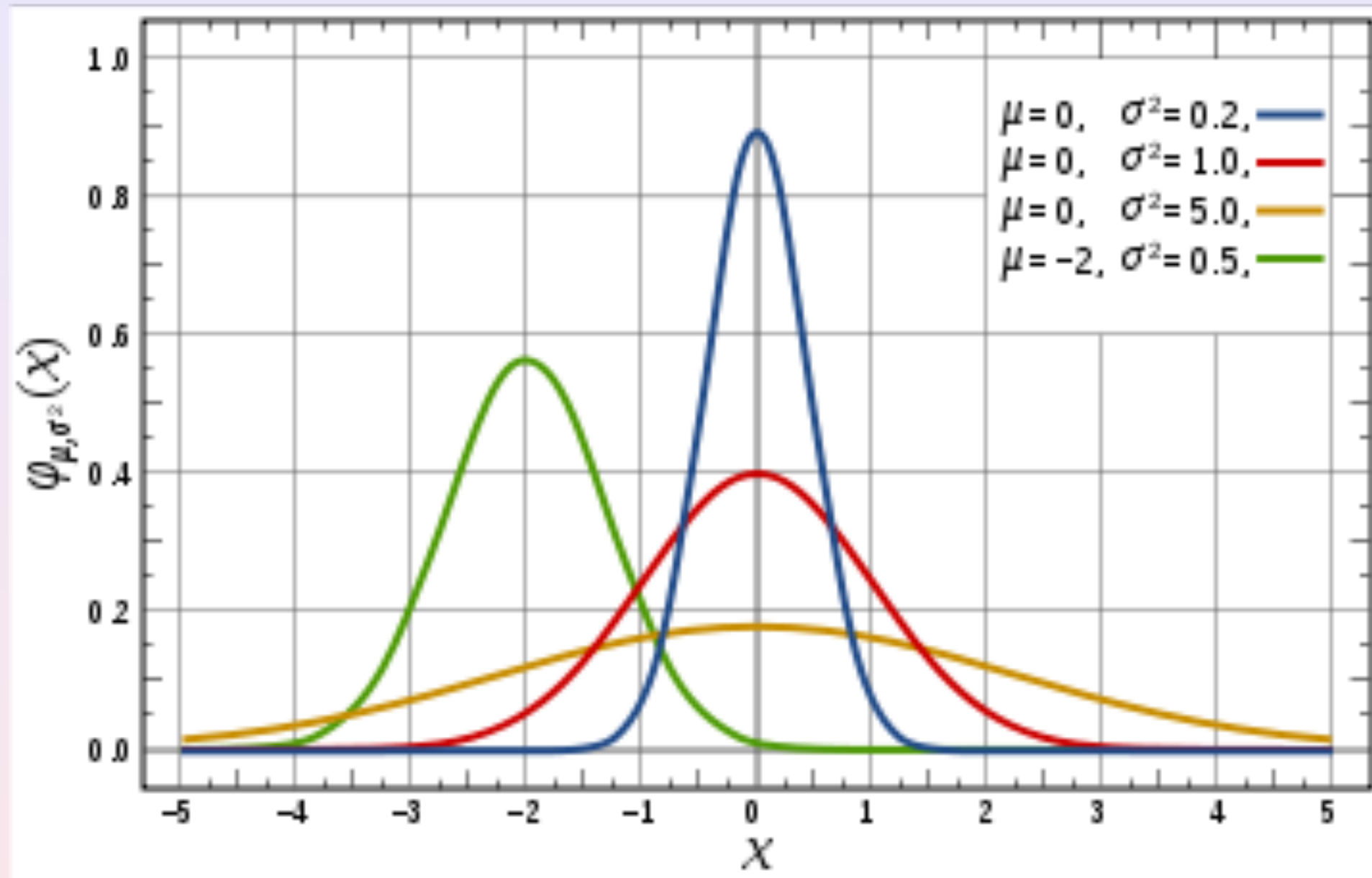


Acoustic noise reduction

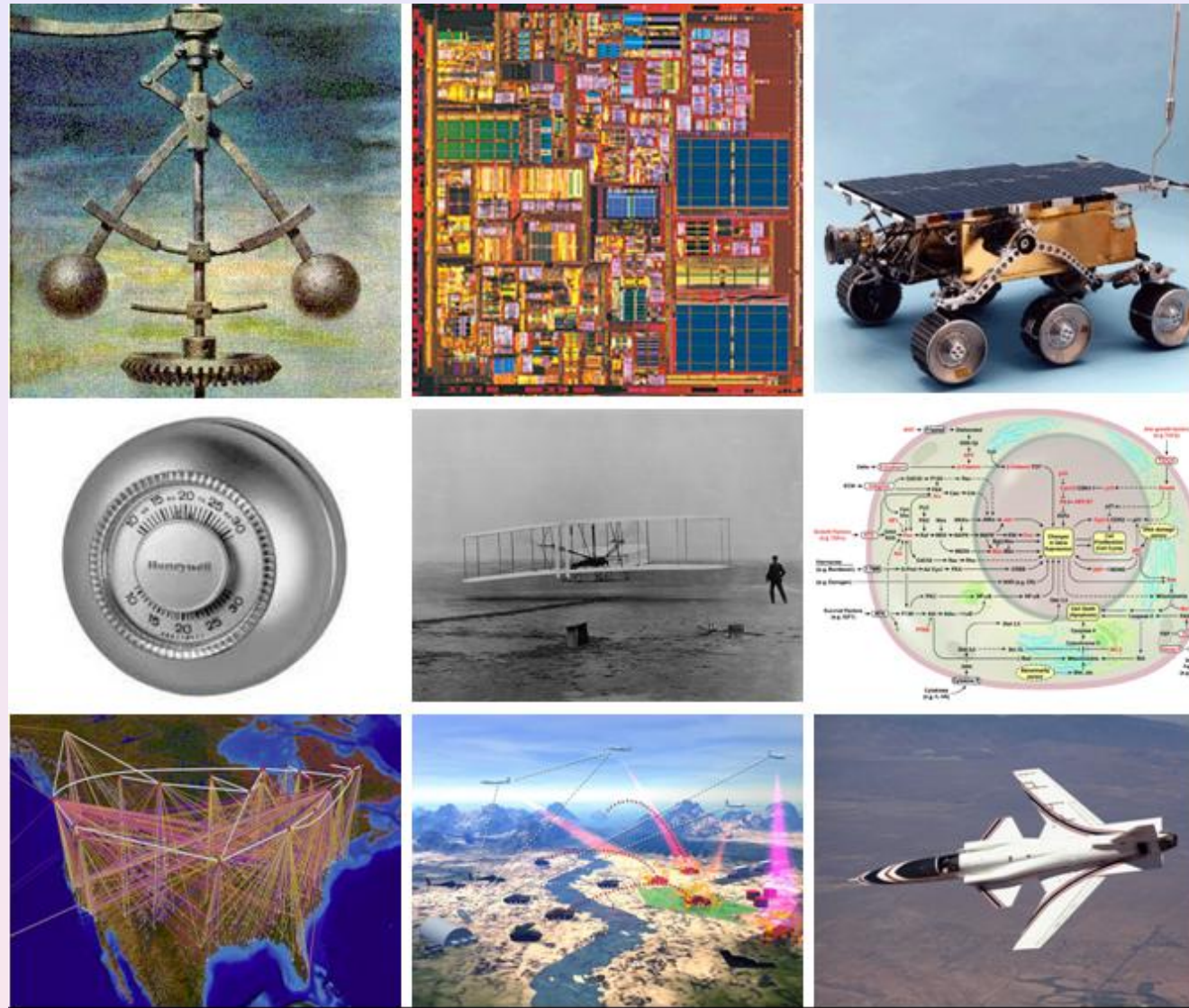
Other applications of noise reduction



Gaussian filters



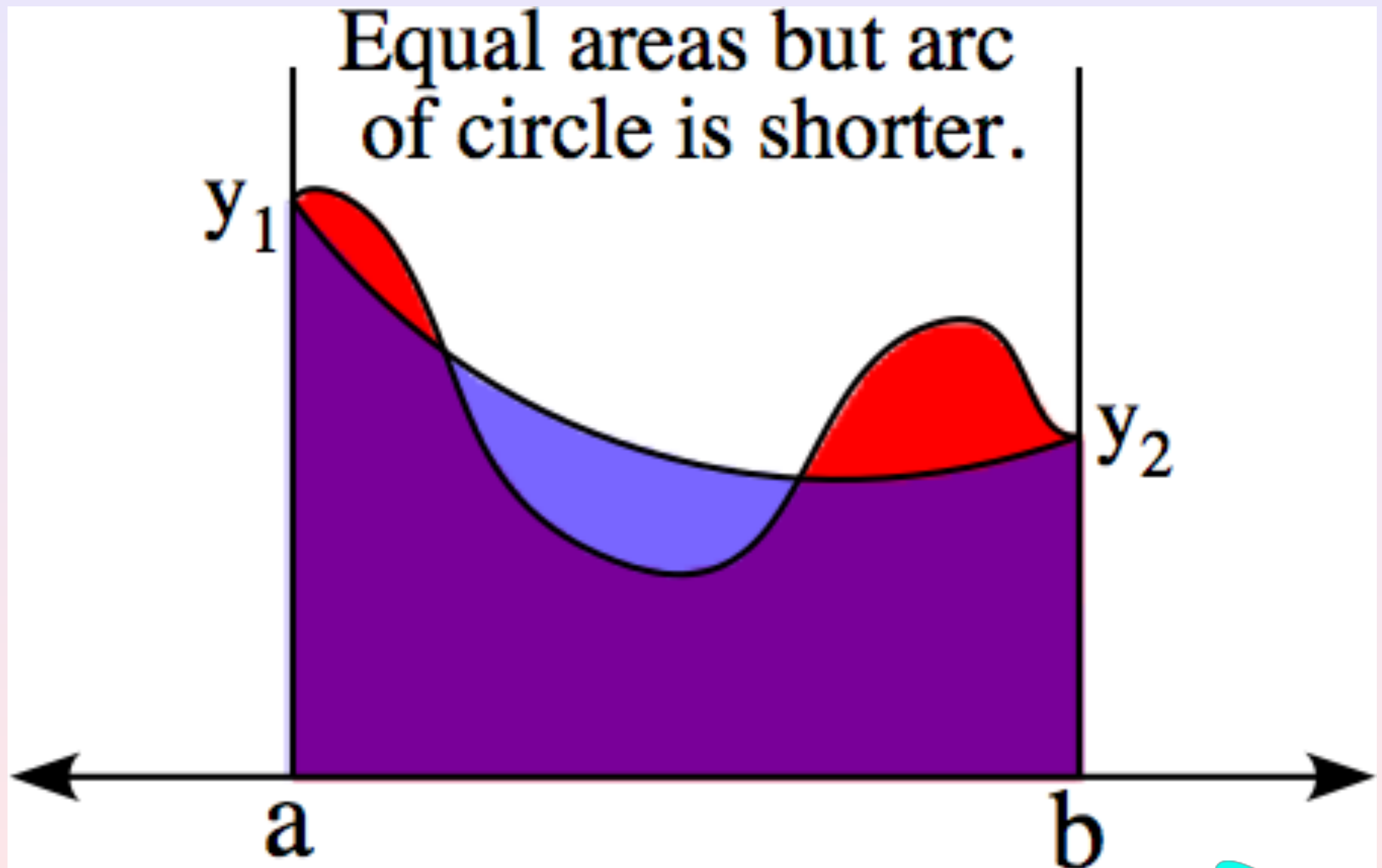
$$u(x) = [G(\cdot) \star f(\cdot)](x); \quad G(x) = (4\pi)^{-N/2} \exp(-|x|^2/4).$$

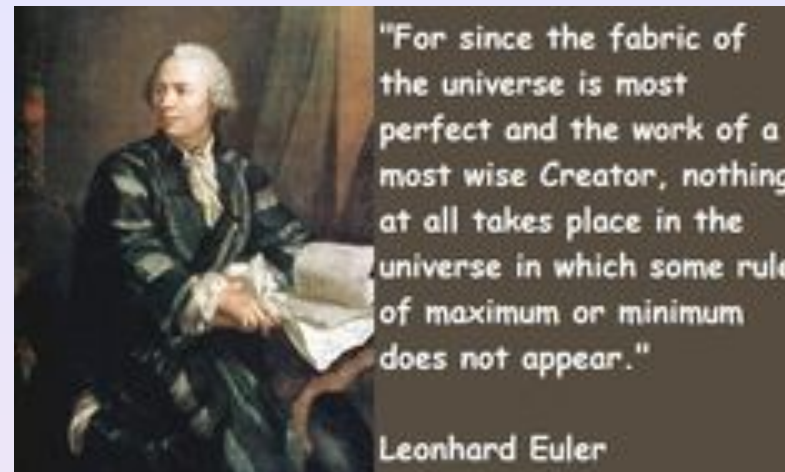


Control in an information rich World, SIAM, R. Murray Ed., 2003.

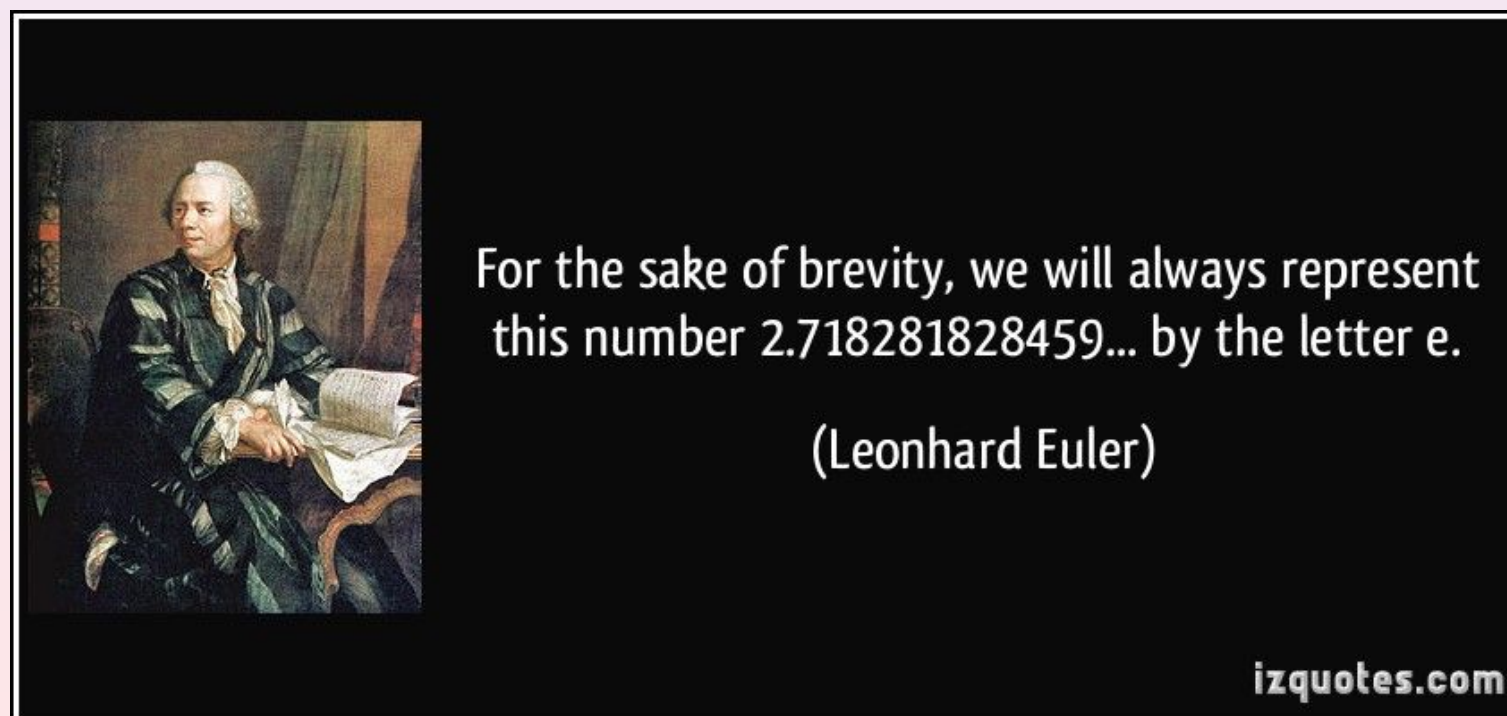
Calculus of Variations

The Calculus of Variations = The search of minimality





Leonhard Euler(1707-1783): For since the fabric of the universe is most perfect and the work of a most wise creator, nothing at all takes place in the universe in which some rule of the maximum or minimum does not appear.



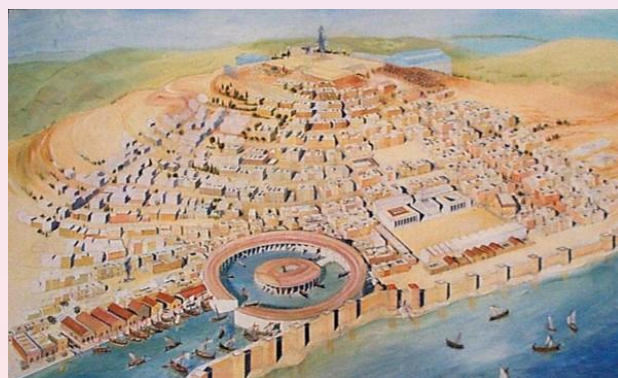
Isoperimetric inequalities

The isoperimetric inequality states, for the length L of a closed curve and the area A of the planar region that it encloses, that

$$4\pi A \leq L^2,$$

and that equality holds if and only if the curve is a circle.

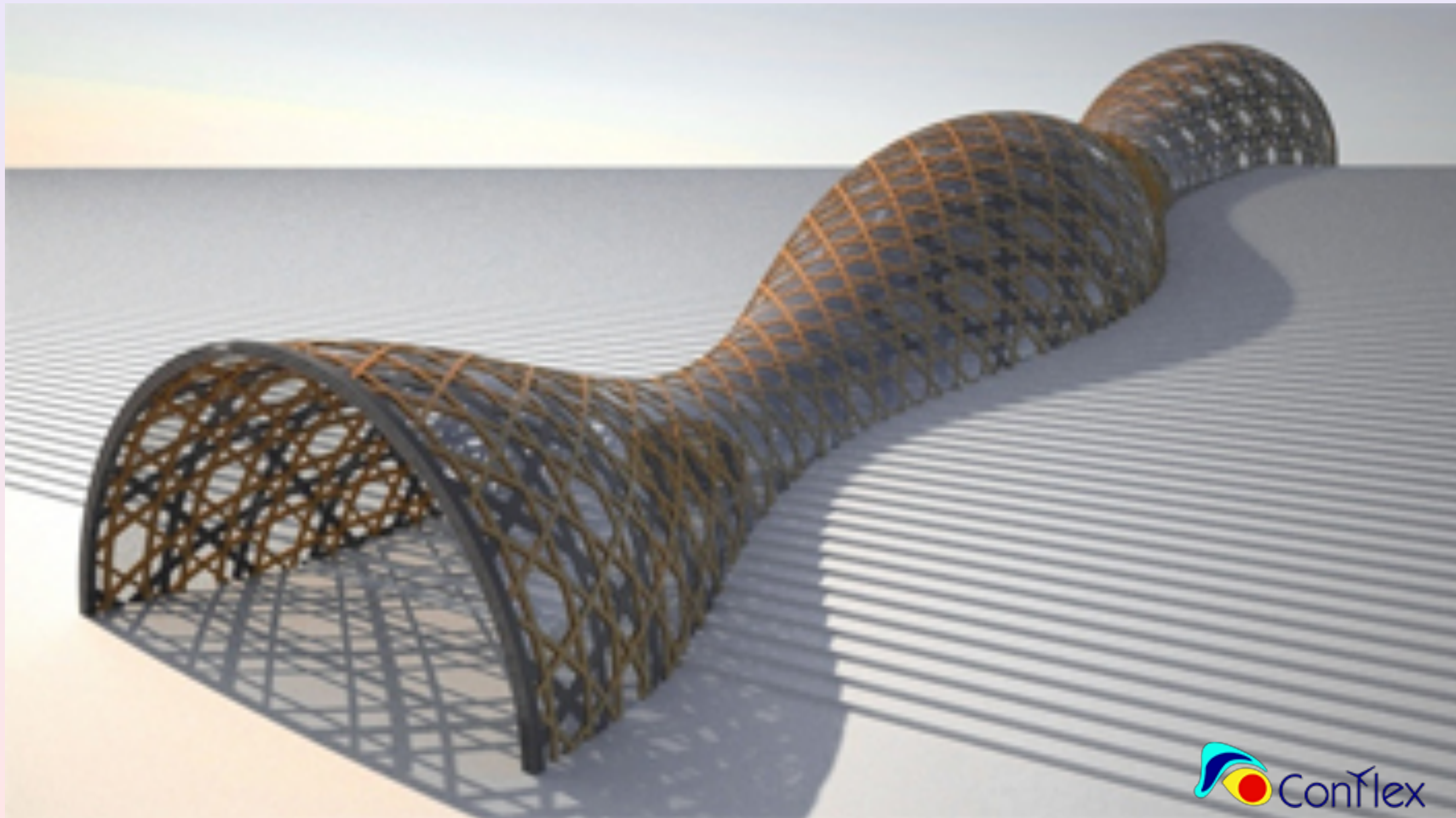
Dido's problem; named after Dido, the legendary founder and first queen of Carthage.

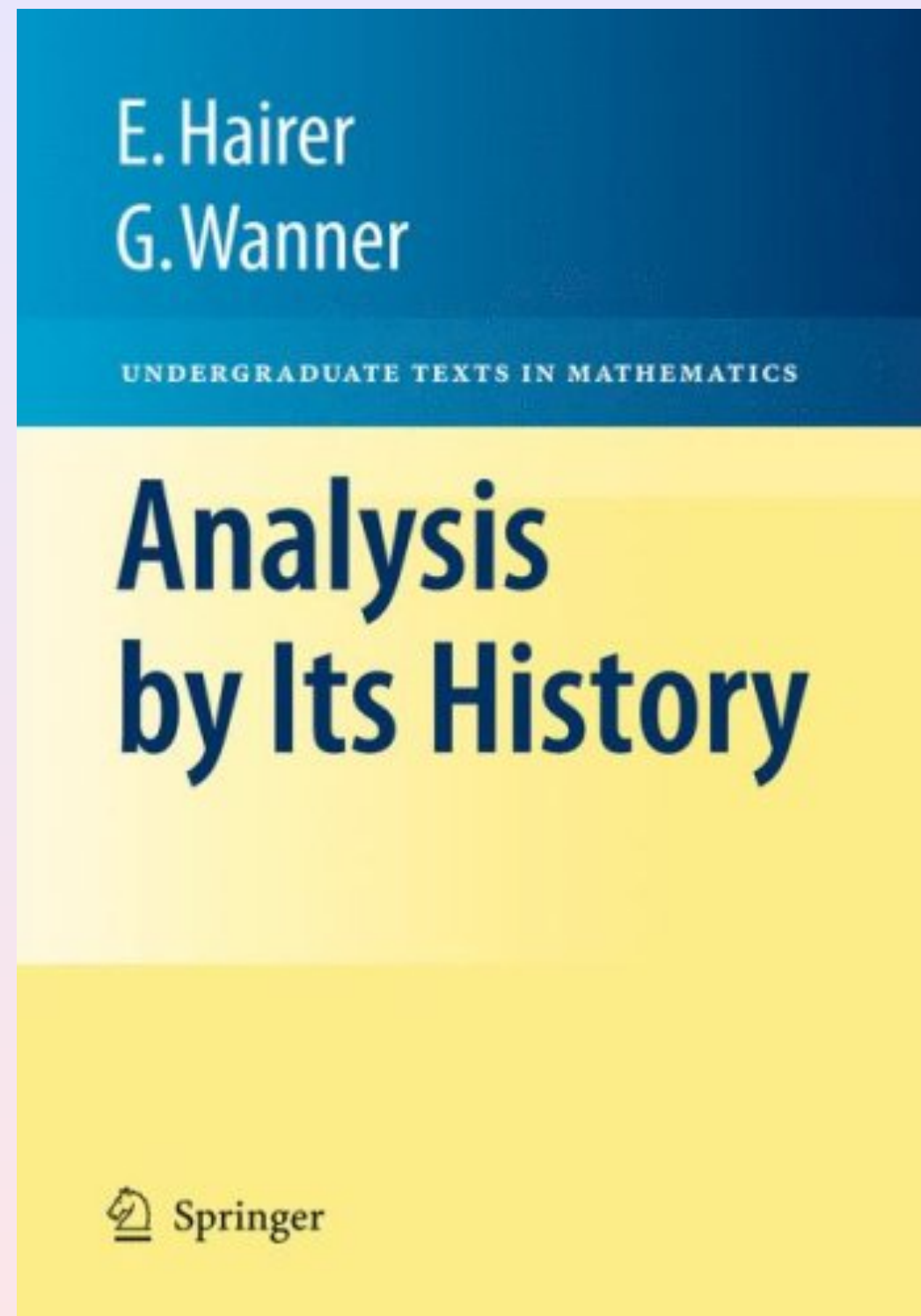


Carthage & Cologne

Geodesic curves

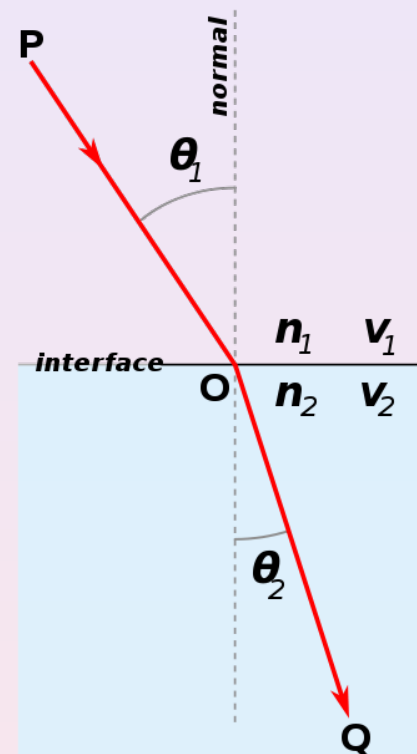
The curve of shortest length, **geodesic**, connecting two points.





Fermat's principle/Snellious law

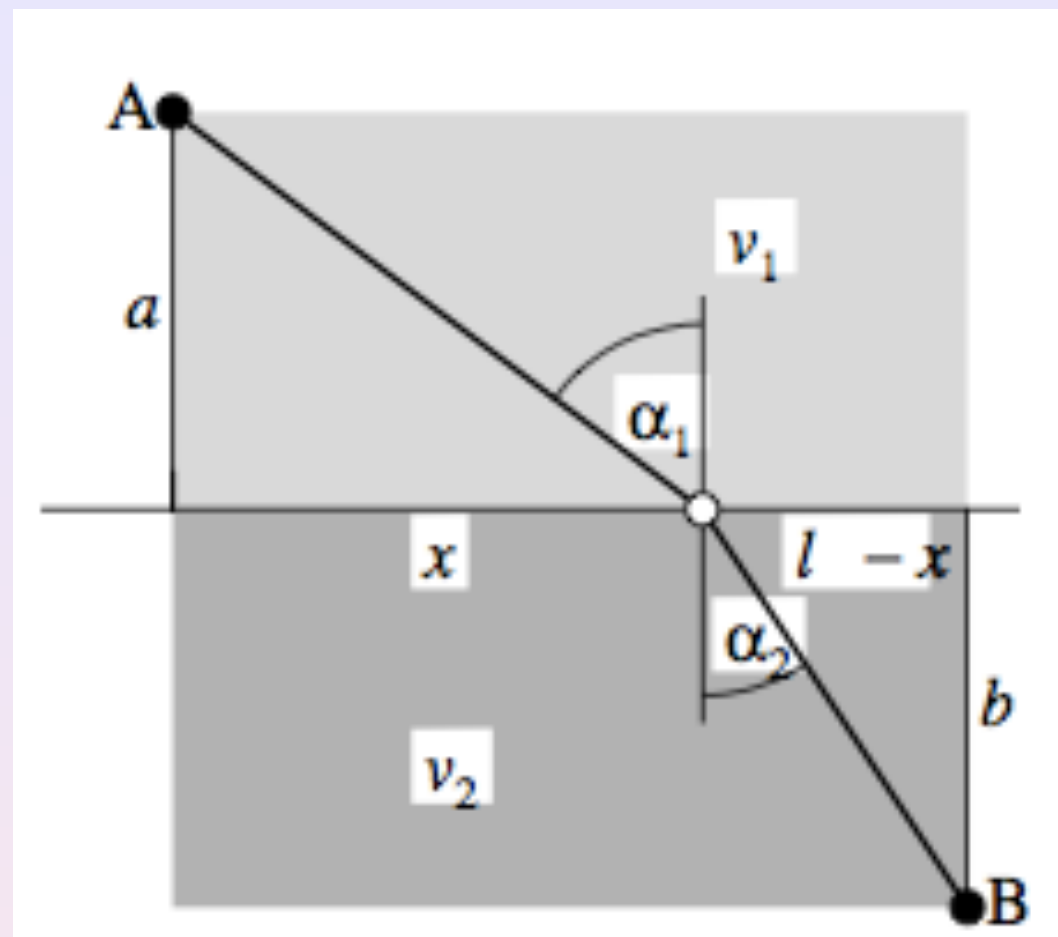
Fermat's principle: light follows the path of shortest optical length connecting two points, where the optical length depends upon the material of the medium.



$$\frac{\sin(\theta_1)}{\sin(\theta_2)} = \frac{v_1}{v_2} = \frac{n_2}{n_1}.$$

Named after the dutch astronomer Willebrord Snellius (1580 - 1626).
 Pierre de Fermat (1601 - 1665).





To find x s. t.

$$T = \frac{\sqrt{a^2 + x^2}}{v_1} + \frac{\sqrt{b^2 + (\ell - x)^2}}{v_2}.$$

Fermat found the problem too difficult for an analytical treatment (**admit that this problem is not one of the easiest**). The computations were then proudly performed by Leibniz (1684)



$$T' = \frac{1}{v_1} \frac{2x}{2\sqrt{a^2 + x^2}} - \frac{1}{v_2} \frac{2(\ell - x)}{2\sqrt{b^2 + (\ell - x)^2}}.$$

Observing that $\sin(\alpha_1) = x/\sqrt{a^2 + x^2}$; $\sin(\alpha_2) = (\ell - x)/\sqrt{b^2 + (\ell - x)^2}$ we see that this derivative vanishes whenever

$$\frac{\sin(\theta_1)}{\sin(\theta_2)} = \frac{v_1}{v_2}.$$

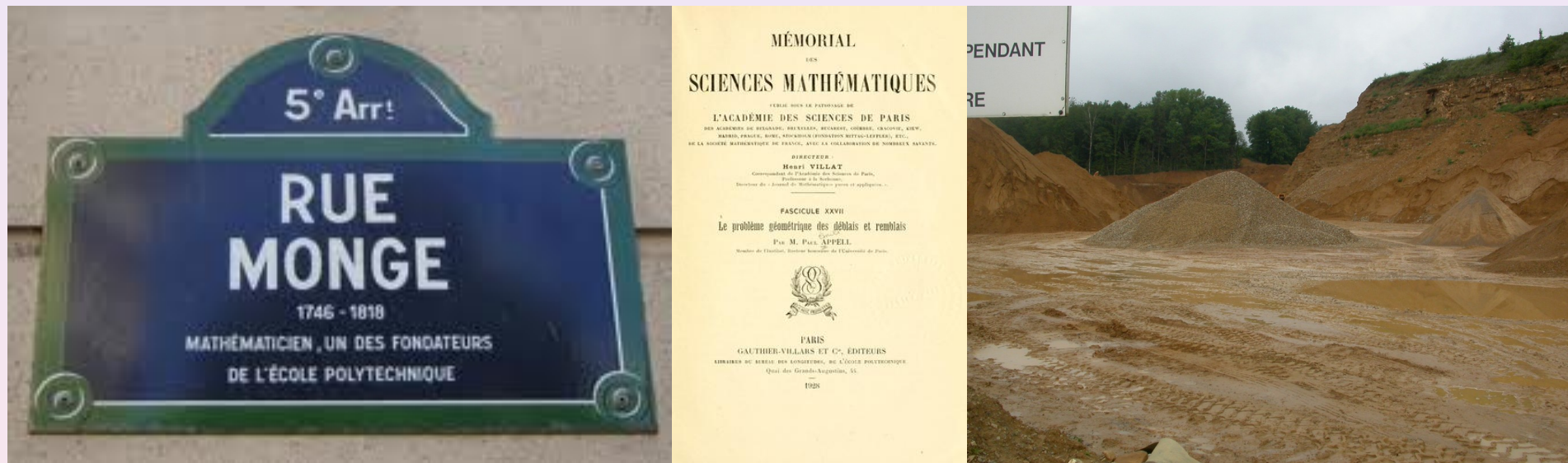
Furthermore:

$$T'' = \frac{1}{v_1} \frac{a^2}{(a^2 + x^2)^{3/2}} + \frac{1}{v_2} \frac{b^2}{(b^2 + (\ell - x)^2)^{3/2}} > 0,$$

showing that the critical point is the minimizer.

Optimal transport

In mathematics and economics, transportation theory refers to the study of optimal transportation and allocation of resources. The problem was formalized by the French mathematician **Gaspard Monge** in 1781 (“Sur la théorie des déblais et des remblais” (Mém. de l’Acad. de Paris, 1781))



But the origins of the potential applications of the idea of optimal transport and geodesic paths goes back to the ancient Egypt where the “harpenodaptai” had as main task drawing long straight lines on the sand of the desert.

Minimal surfaces

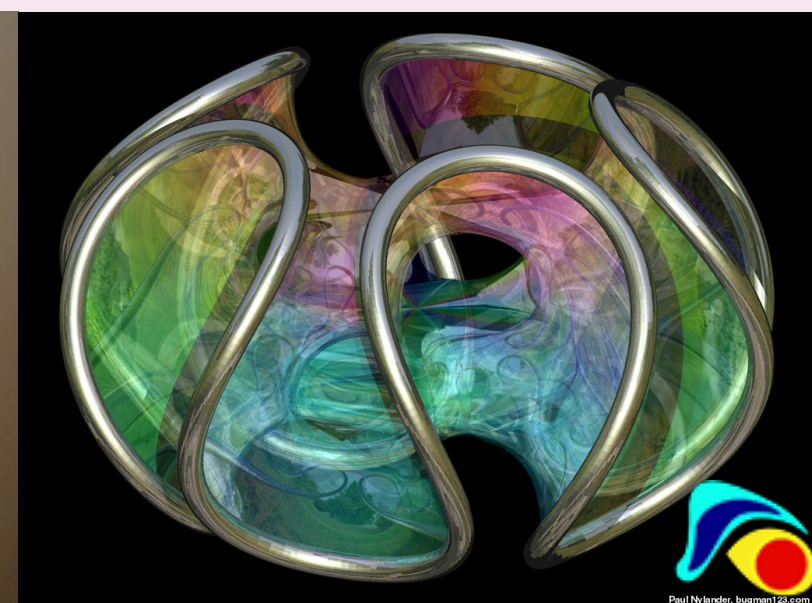
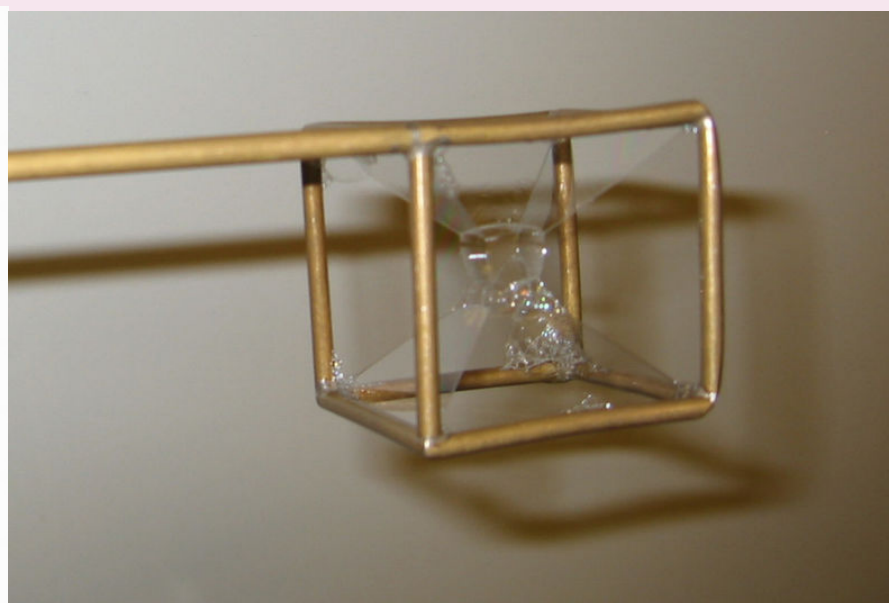
Minimal surfaces are defined as surfaces with zero mean curvature. Finding a minimal surface of a boundary with specified constraints is a problem in the Calculus of Variations and is sometimes known as

Plateau's problem.

Physical models of area-minimizing minimal surfaces can be made by dipping a wire frame into a soap solution, forming a soap film, which is a minimal surface whose boundary is the wire frame.

Enneper's surface:

$$x = u(1 - u^2/3 + v^2)/3; y = -v(1 - v^2/3 + u^2)/3; z = (u^2 - v^2)/3.$$



ConFlex

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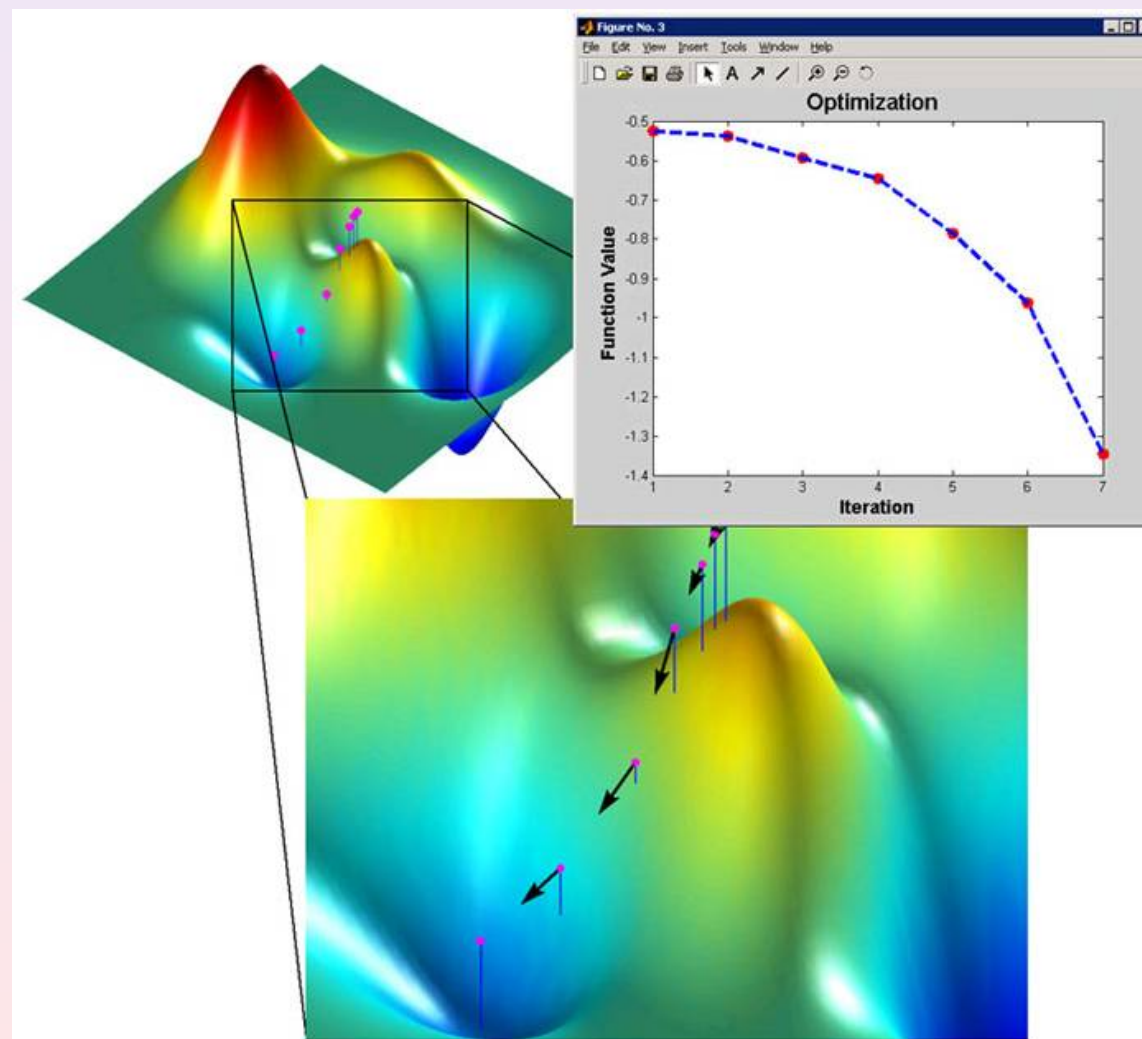
The computer perspective

Computational minimization

$$J(u^*) = \min_{u \in \mathcal{U}} J(u).$$

Gradient methods:

$$u_{k+1} = u_k - \rho \nabla J(u_k).$$



Montecarlo methods

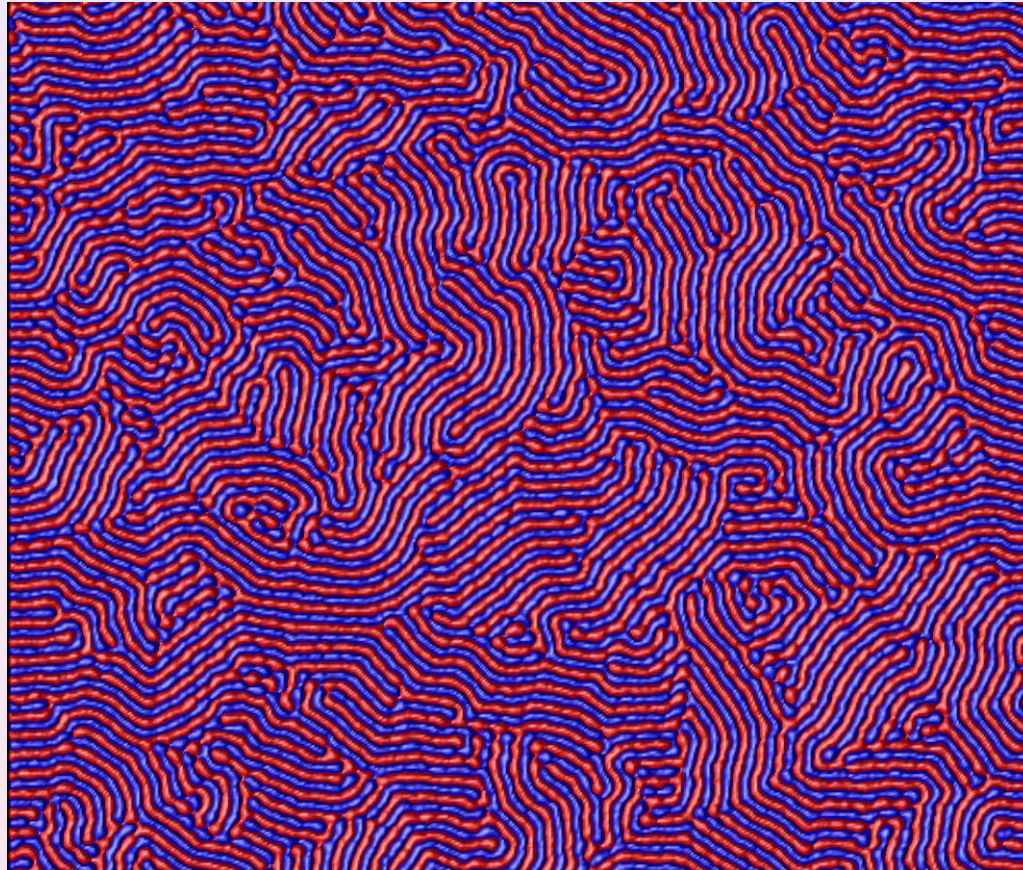
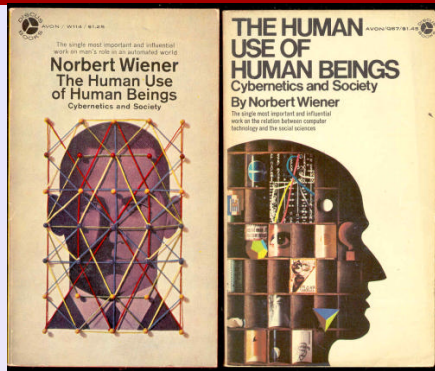


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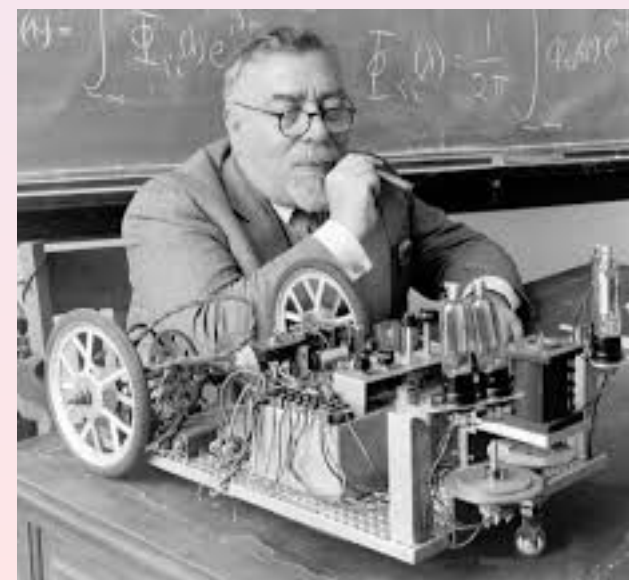
Cybernetics



“Cybernétique” was proposed by the French physicist **A.-M. Ampère** in the XIX Century to design the nonexistent science of process controlling. This was quickly forgotten until 1948, when **Norbert Wiener** (1894–1964) chose “**Cybernetics**” as the title of his famous book.

Wiener defined Cybernetics as “ **the science of control and communication in animals and machines**” .

In this way, he established the connection between Control Theory and Physiology and anticipated that, in a desirable future, engines would obey and imitate human beings.



Robotic arm



Let $n, m \in \mathbb{N}^*$ and $T > 0$ and consider the following linear finite-dimensional system

$$x'(t) = Ax(t) + Bu(t), \quad t \in (0, T); \quad x(0) = x^0. \quad (1)$$

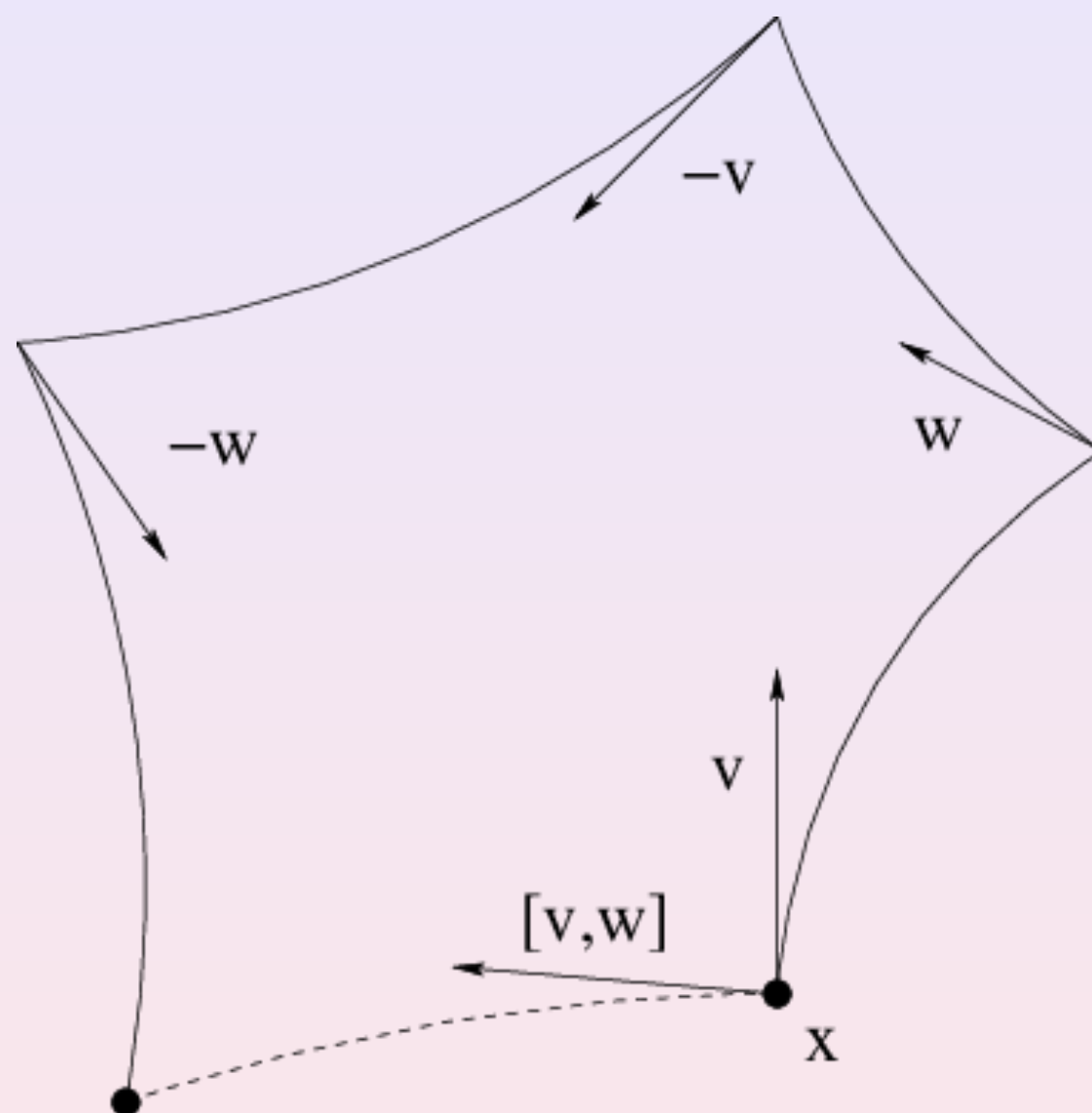
In (??), A is a $n \times n$ real matrix, B is of dimensions $n \times m$ and x^0 is the initial state of the system in \mathbb{R}^n . The function $x : [0, T] \rightarrow \mathbb{R}^n$ represents the *state* and $u : [0, T] \rightarrow \mathbb{R}^m$ the *control*.

¿Can we control the state x of n components with only m controls, even if $n \gg m$?

Theorem

(1958, Rudolf Emil Kálmán (1930–)) System is controllable iff

$$\text{rank}[B, AB, \dots, A^{n-1}B] = n.$$



Proof:

From the variation of constants formula:

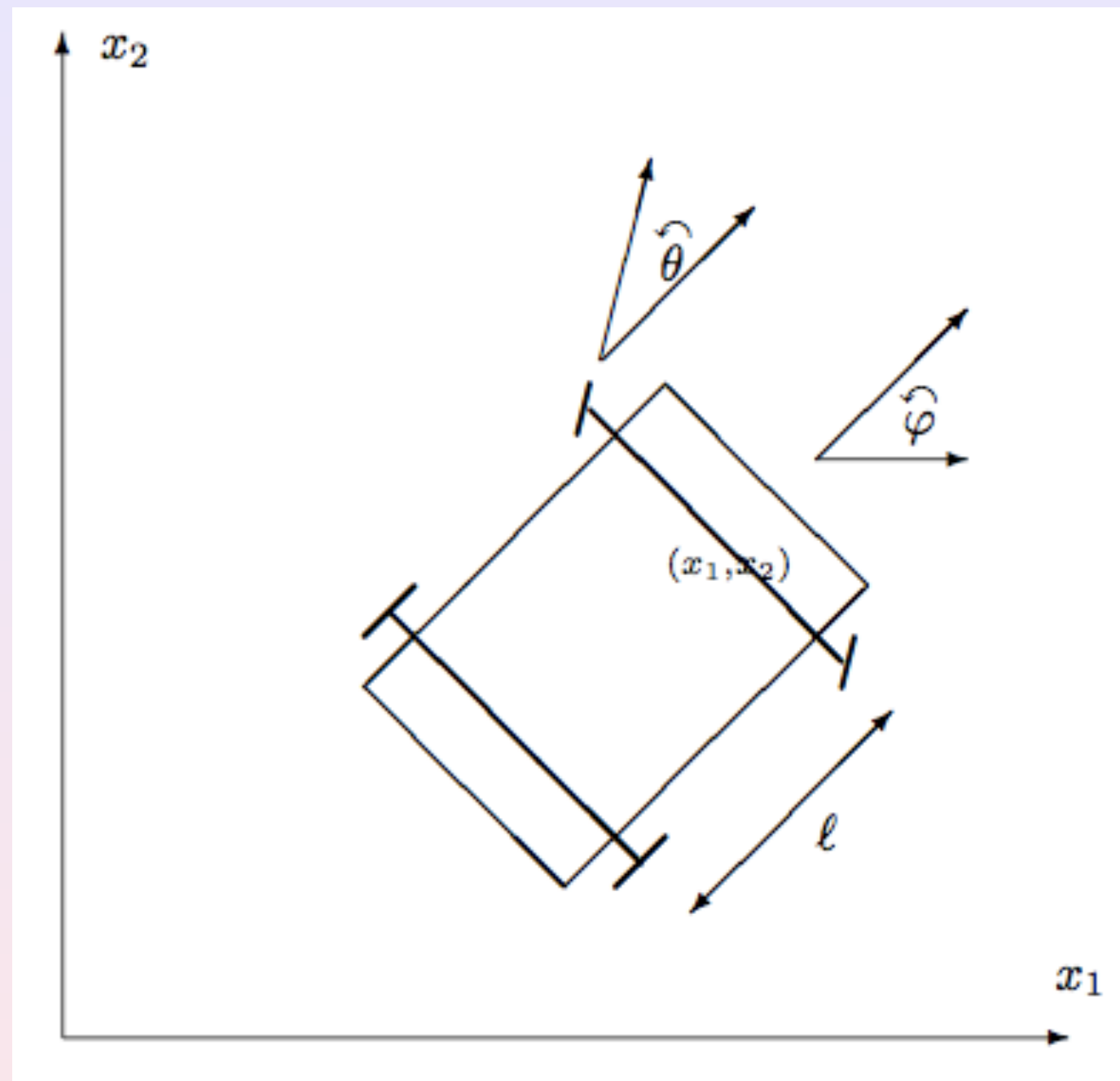
$$x(t) = e^{At}x^0 + \int_0^t e^{A(t-s)}Bu(s)ds = e^{At}x^0 + \int_0^t \sum_{k \geq 0} \frac{(t-s)^k}{k!} A^k Bu(s)ds.$$

By Cayley¹-Hamilton's ² theorem A^k for $k \geq n$ is a linear combination of I, A, \dots, A^{n-1} .

¹Arthur Cayley (UK, 1821 - 1895)

²William Rowan Hamilton (Ireland, 1805 - 1865)

An example: Nelson's car.



Two controls suffice to control a four-dimensional dynamical system.

E. Sontag, *Mathematical control theory*, 2nd ed., Texts in Applied Mathematics, vol.6, Springer-Verlag, New York, 1998.

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Optimal design

The optimal pancake

MailOnline

The perfect pancake? Easy, just follow this formula ... $100 - [10L - 7F + C(k - C) + T(m - T)] / (S - E)$

By [Daily Mail Reporter](#)

Last updated at 9:49 AM on 24th February 2009

With Shrove Tuesday tomorrow it was perhaps inevitable that an eager scientist would apply their skills to creating the perfect pancake.

Maths expert Dr Ruth Fairclough stepped up to the challenge, unveiling a complex algebra formula to help chefs nail the dish on the day.

The 34-year-old senior lecturer of mathematics and statistics worked out the food formula because her two daughters loved eating pancakes so much.

Dr Ruth, who teaches at Wolverhampton University found that $100 - [10L - 7F + C(k - C) + T(m - T)] / (S - E)$ created the tastiest snack.

In the complex formula L represents the number of lumps in the batter and C equals its consistency.

The letter F stands for the flipping score, k is the ideal consistency and T is the temperature of the pan.

Ideal temp of pan is represented by m, S is the length of time the batter stands before cooking and E is the length of time the cooked pancake sits before being eaten.



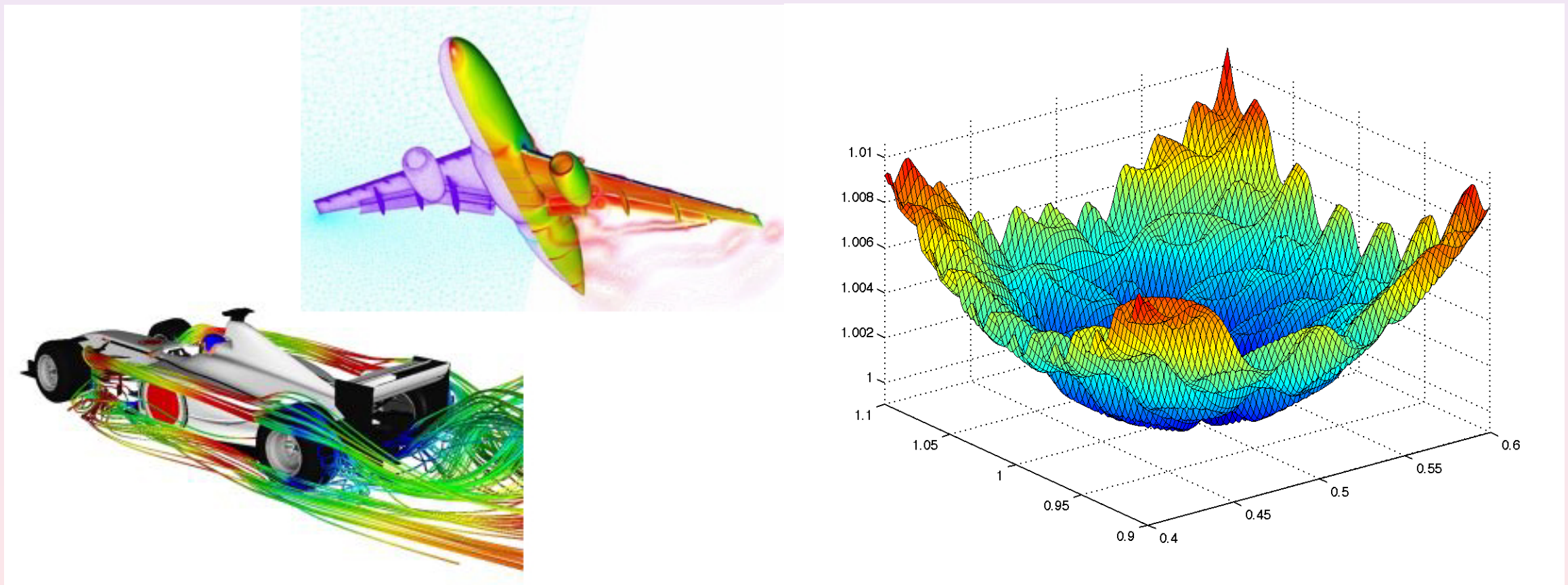
Optimal shape design in aeronautics.

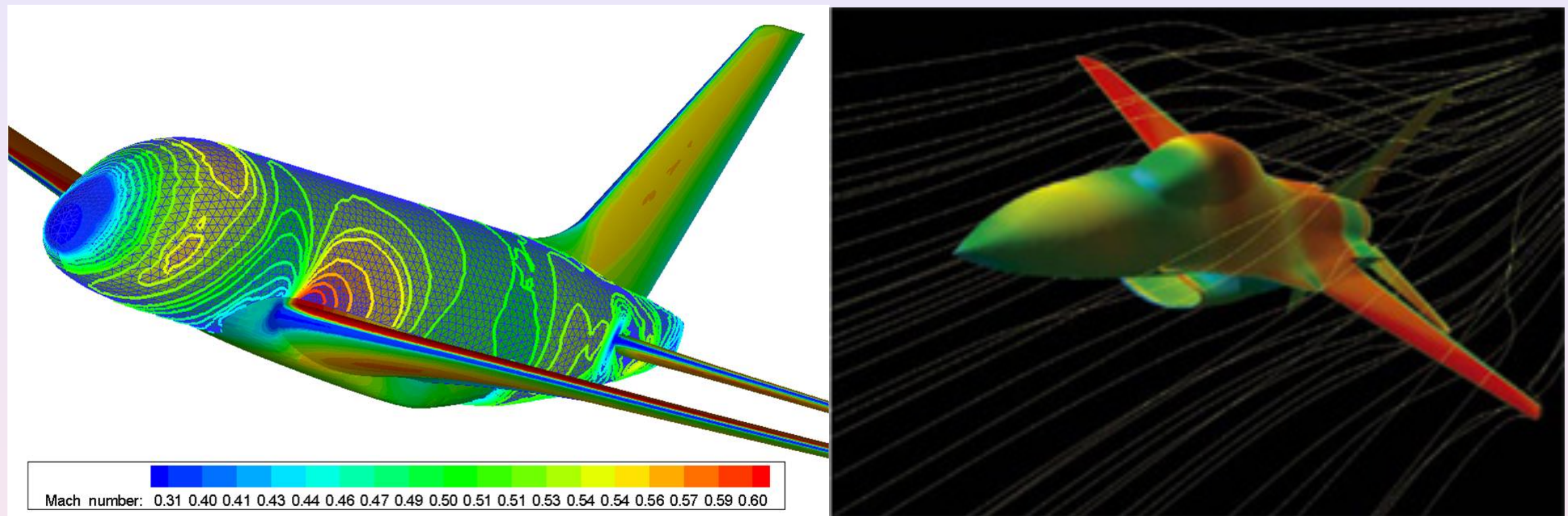
- **Objective:** To modify the shape of the airplane so to improve its efficiency, security, reduce noise, energy consumption, reduce drag, augment lift,...
- **Point of view:** That of the wind tunnel. The airplane is fixed while air is flowing around.
- **Variations:** When modifying the shape of the airplane, the way air is flowing around is modified, and the pressure field it applies into the airplane as well. The aerodynamical properties of the airplane are modified.



Tools.

- **Computational fluid mechanics:** It allows to simulate the flow of air around a cavity.
- **Optimization:** It allows building an iterative algorithm to improve performance.





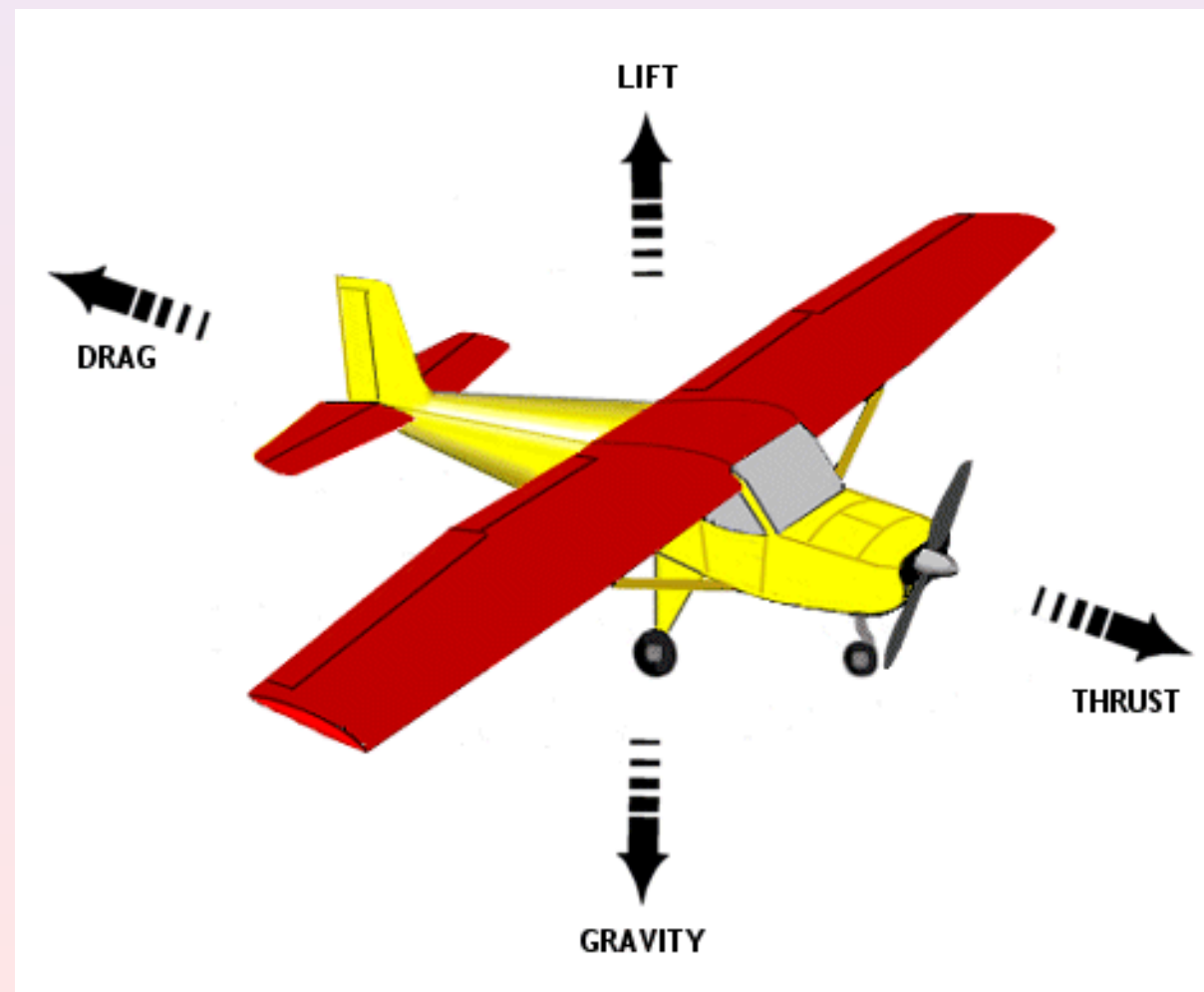
Computed pressure field over the surface of the airplane and flow lines of particles of air.

The method consists on formulating the problem in the context of the Calculus of Variations. To minimize

$$J(\Omega^*) = \min_{\Omega \in \mathcal{C}_{ad}} J(\Omega)$$

where \mathcal{C}_{ad} is the class of **admissible shapes** Ω , and $J =$ is the **cost functional** measuring the efficiency of the design (drag, lift,...)

J depends on Ω but not directly, rather thorough $u(\Omega)$, the solution of the air-dynamics in the exterior of the airplane.



Leonhard Euler



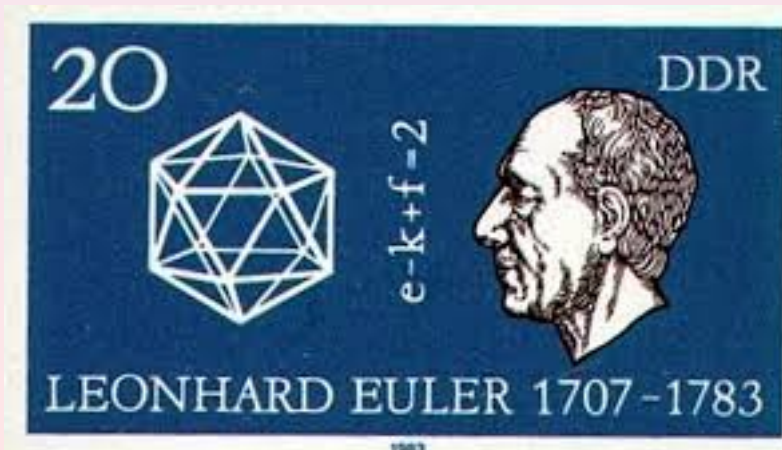
(1707-1783) derived the equations for the motion of perfect fluids, in the absence of viscosity:

$$u_t + u \cdot \nabla u = \nabla p.$$

But D'Alembert observed that the flight of birds would be impossible according to that model.

Claude Louis Marie Henri Navier (1785-1836) and Sir George Gabriel Stokes (1819-1903) much later incorporated the viscosity term:

$$u_t - \nu \Delta u + u \cdot \nabla u = \nabla p.$$

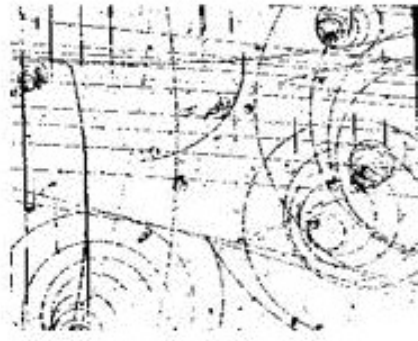


There are many open complex problems in the field of Fluid Mechanics.

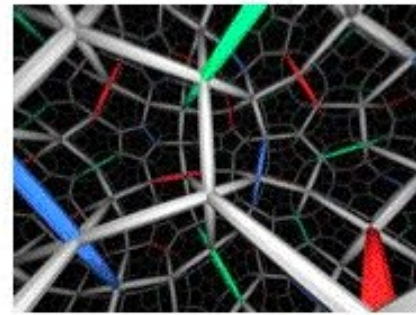


Fluid Mechanics is one of the most important areas of Physics because of its impact on our life : air, water, blood,...

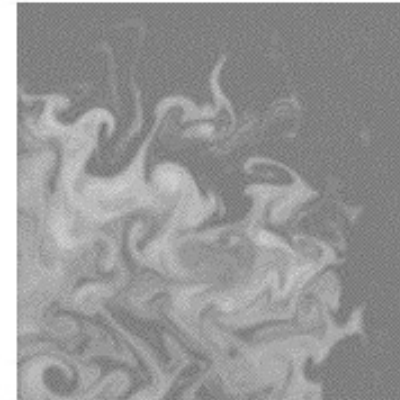




Yang-Mills and Mass Gap



Poincaré Conjecture



Navier-Stokes Equation

Birch and Swinnerton-Dyer
Conjecture

Riemann Hypothesis



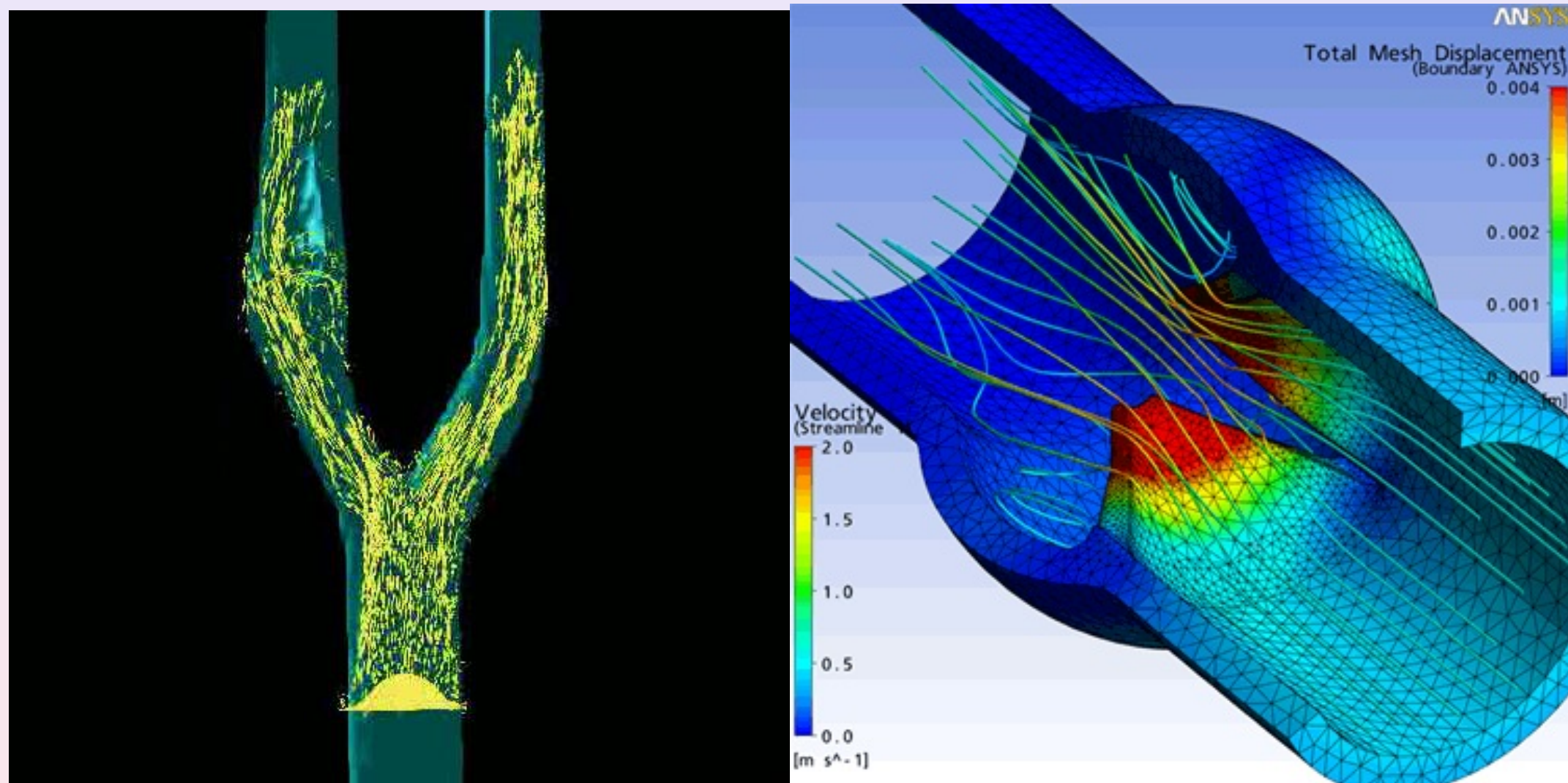
P vs NP Problem

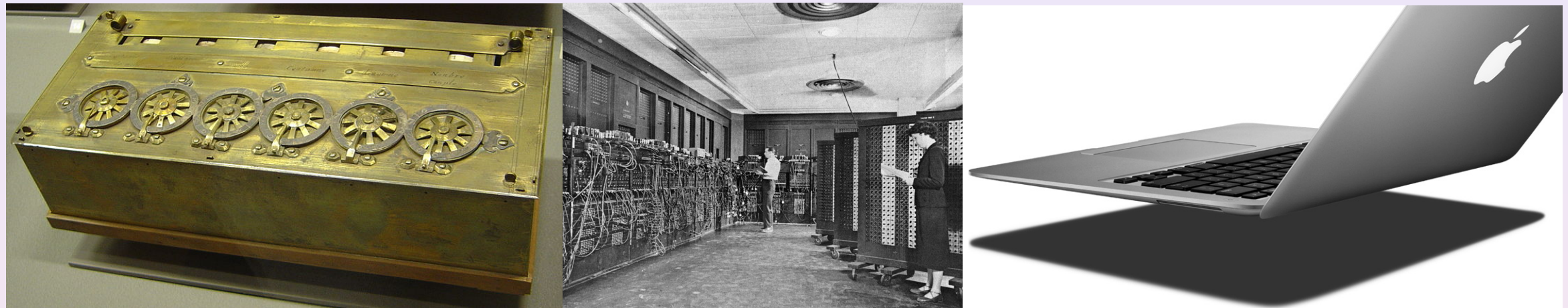


Hodge Conjecture

The millenium problems

The science program is still ongoing to a large extent thanks to computers.





[Pascaline](#), Blaise Pascal, 1645; [ENIAC](#): Electronic Numerical Integrator And Computer, 1946; [Macbook Air](#), 2008.

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Optimisation

In mathematics, computational science, or management science, mathematical optimization (alternatively, optimization or mathematical programming) refers to the selection of a best element from some set of available alternatives.

- Convex programming
- Linear programming
- Semidefinite programming
- Conic programming
- Stochastic programming
- Robust programming
- Combinatorial optimization
- Dynamic programming
- Heuristics and metaheuristics
-

An example in logistics

This is a typical and ubiquitous example in linear programming. A company s_i , $i \leq 1 \leq M$ items in each of the M storage locations. N clients request r_j items each, $1 \leq j \leq N$. The cost of transportation between the i -th storage location and the j -th client is c_{ij} . We have to decide about the number of items to be delivered from the i -th storage location the j -th client, v_{ij} .

Of course we want to minimize the cost of transportation. The problem is then that of minimizing the functional

$$\inf_{\{v_{ij}\}} \left(\sum_{i=1}^M \sum_{j=1}^N c_{ij} v_{ij} \right)$$

under the constraints

$$v_{ij} \geq 0; \sum_{j=1}^N v_{ij} \leq s_i; \sum_{i=1}^M v_{ij} = r_j, 1 \leq i \leq M; 1 \leq j \leq N.$$



These tools are so much used that nowadays there is plenty of software available both free and commercial: IPOPT

Last modified on 08/08/11 20:01:00

Welcome to the Ipopt home page

Note that these project webpages are based on Wiki, which allows webusers to modify the content to correct typos, add information, or share their experience and tips with other users. You are welcome to contribute to these project webpages. To edit these pages or submit a ticket you must first [register and login](#).

Introduction

Ipopt (**I**nterior **P**oint **OPT**imizer, pronounced eye-pea-Opt) is a software package for large-scale [nonlinear optimization](#). It is designed to find (local) solutions of mathematical optimization problems of the form

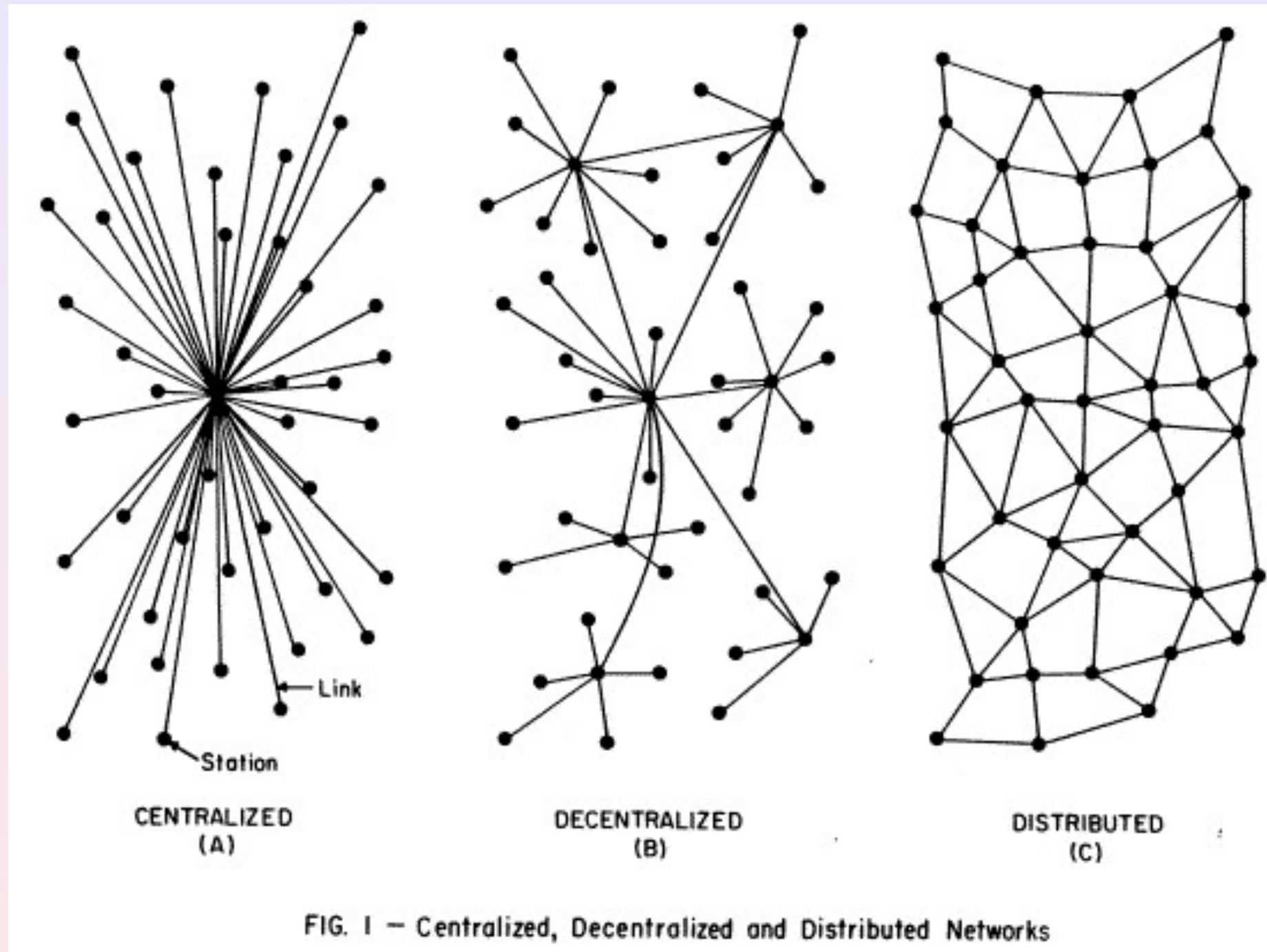
$$\begin{array}{ll} \min & f(x) \\ x \text{ in } & \mathbb{R}^n \\ \text{s.t.} & g_L \leq g(x) \leq g_U \\ & x_L \leq x \leq x_U \end{array}$$

where $f(x): \mathbb{R}^n \rightarrow \mathbb{R}$ is the objective function, and $g(x): \mathbb{R}^n \rightarrow \mathbb{R}^m$ are the constraint functions. The vectors g_L and g_U denote the lower and upper bounds on the constraints, and the vectors x_L and x_U are the bounds on the variables x . The functions $f(x)$ and $g(x)$ can be nonlinear and nonconvex, but should be twice continuously differentiable. Note that equality constraints can be formulated in the above formulation by setting the corresponding components of g_L and g_U to the same value.

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Future challenges

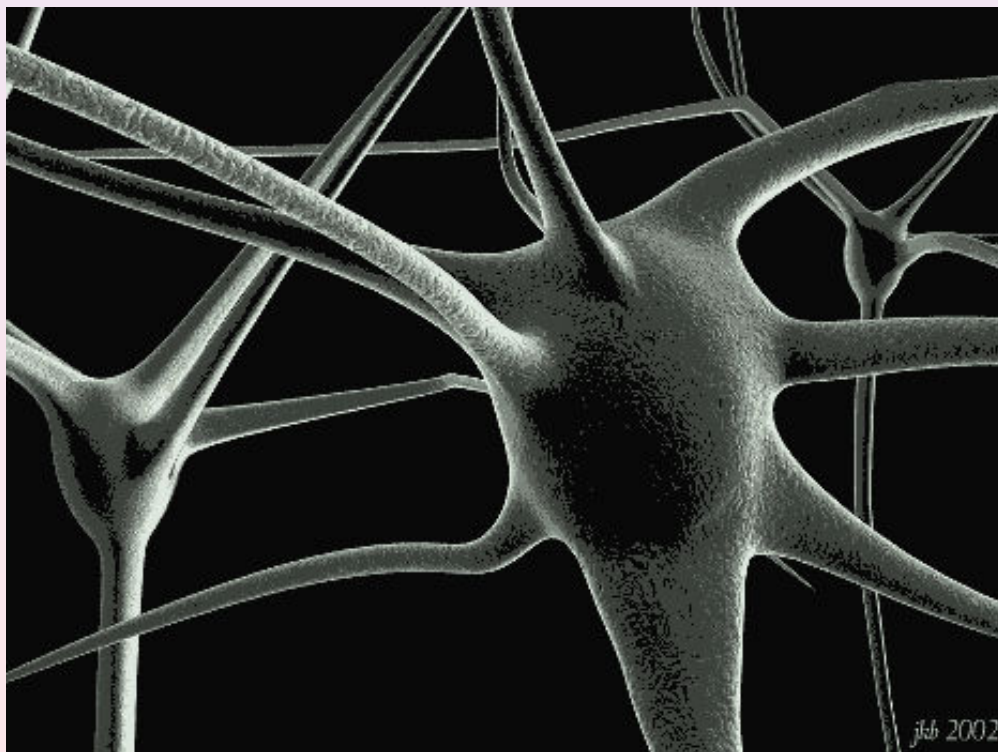
We live more and more on a complex network



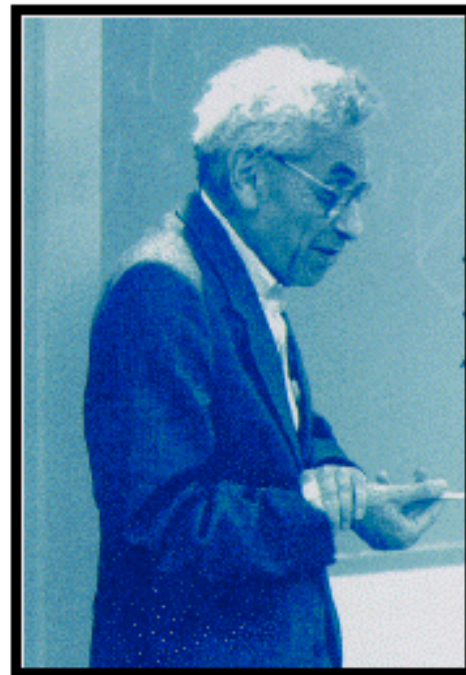
Mathematics are and will be increasingly influenced by the challenge of dealing with **complexity** and **multidisciplinarity**. The following areas will gain relevance:

- Discrete mathematics, combinatorics, graphs,...;
- Data mining/Big data;
- Statistical learning;

and other fields of research such as **neurosciences** and **social sciences**.



A mathematician is a machine for turning coffee into theorems



The Erdős Number Project

This is the website for the Erdős Number Project, which studies research collaboration among mathematicians.

The site is maintained by **Jerry Grossman** at **Oakland University**. **Patrick Ion**, a retired editor at **Mathematical Reviews**, and **Rodrigo De Castro** at the **Universidad Nacional de Colombia, Bogota** provided assistance in the past. Please address all comments, additions, and corrections to Jerry at grossman@oakland.edu.

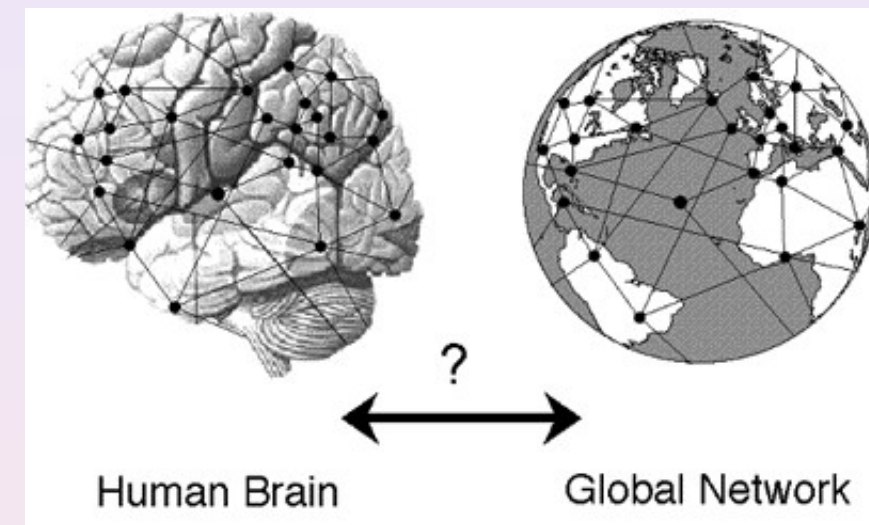
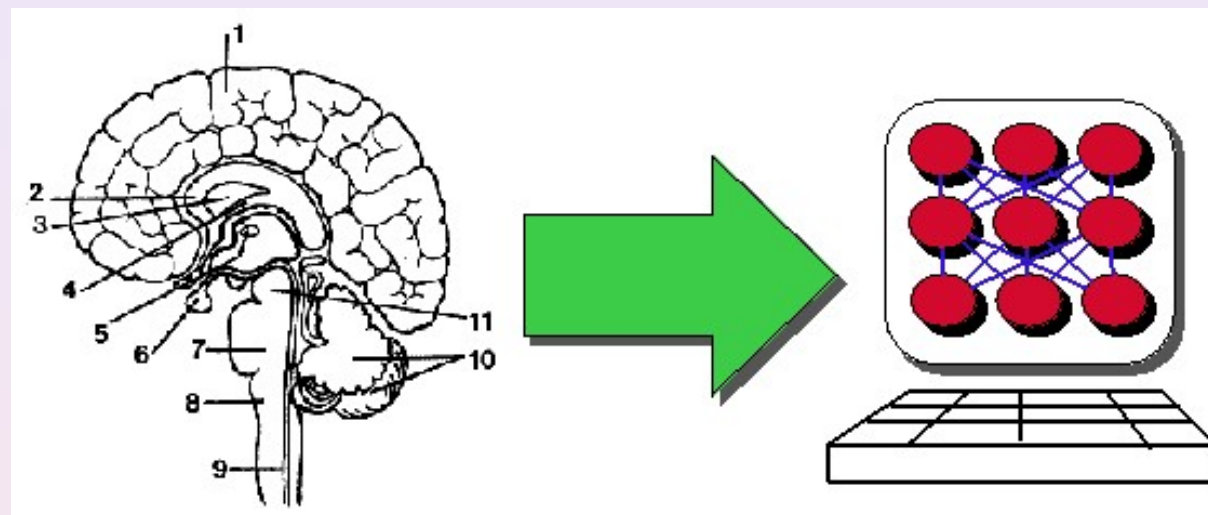
Erdős numbers have been a part of the **folklore of mathematicians** throughout the world for many years. For an introduction to our project, a description of what Erdős numbers are, what they can be used for, who cares, and so on, choose the "What's It All About?" link below. To find out who **Paul Erdős** is, look at this **biography** at the MacTutor History of Mathematics Archive, or choose the "Information about Paul Erdős" link below. Some useful information can also be found in **this Wikipedia article**, which may or may not be totally accurate.

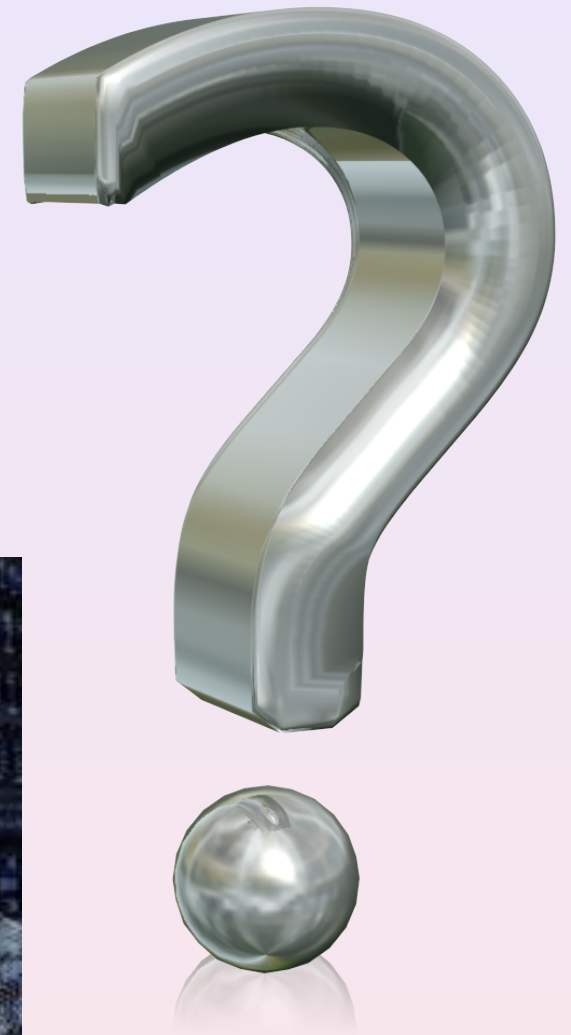
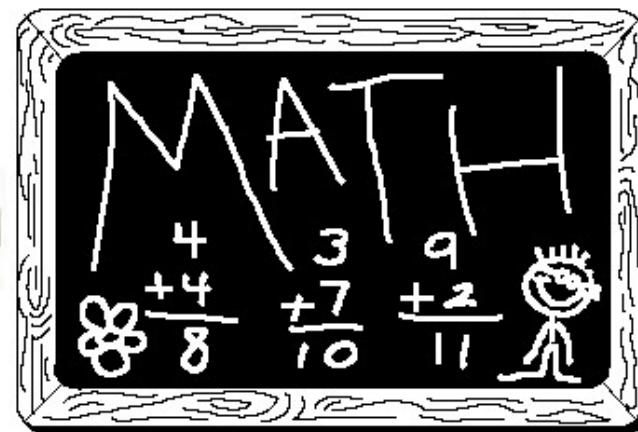
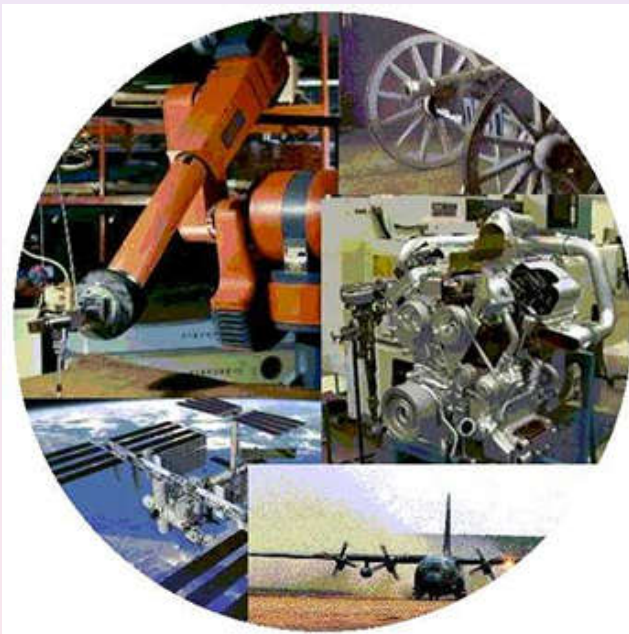
Paul Erdős (1913–1996) was a Hungarian mathematician. He published more papers than any other mathematician in history, working with hundreds of collaborators. He worked on problems in combinatorics, graph theory, number theory, classical analysis, approximation theory, set theory, and probability theory.



There will be unexpected advances in computing algorithms...

| Year | Development | Key early figures |
|------|----------------------------------|--|
| 263 | Gaussian elimination | Liu, Lagrange, Gauss, Jacobi |
| 1671 | Newton's method | Newton, Raphson, Simpson |
| 1795 | Least-squares fitting | Gauss, Legendre |
| 1814 | Gauss quadrature | Gauss, Jacobi, Christoffel, Stieltjes |
| 1855 | Adams ODE formulas | Euler, Adams, Bashforth |
| 1895 | Runge-Kutta ODE formulas | Runge, Heun, Kutta |
| 1910 | Finite differences for PDE | Richardson, Southwell, Courant, von Neumann, Lax |
| 1936 | Floating-point arithmetic | Torres y Quevedo, Zuse, Turing |
| 1943 | Finite elements for PDE | Courant, Feng, Argyris, Clough |
| 1946 | Splines | Schoenberg, de Casteljau, Bezier, de Boor |
| 1947 | Monte Carlo simulation | Ulam, von Neumann, Metropolis |
| 1947 | Simplex algorithm | Kantorovich, Dantzig |
| 1952 | Lanczos and CG iterations | Lanczos, Hestenes, Stiefel |
| 1952 | Stiff ODE solvers | Curtiss, Hirschfelder, Dahlquist, Gear |
| 1954 | Fortran | Backus |
| 1958 | Orthogonal linear algebra | Aitken, Givens, Householder, Wilkinson, Golub |
| 1959 | Quasi-Newton iterations | Davidon, Fletcher, Powell, Broyden |
| 1961 | QR algorithm for eigenvalues | Rutishauser, Kublanovskaya, Francis, Wilkinson |
| 1965 | Fast Fourier transform | Gauss, Cooley, Tukey, Sande |
| 1971 | Spectral methods for PDE | Chebyshev, Lanczos, Clenshaw, Orszag, Gottlieb |
| 1971 | Radial basis functions | Hardy, Askey, Duchon, Micchelli |
| 1973 | Multigrid iterations | Fedorenko, Bakhvalov, Brandt, Hackbusch |
| 1976 | EISPACK, LINPACK, LAPACK | Moler, Stewart, Smith, Dongarra, Demmel, Bai |
| 1976 | Nonsymmetric Krylov iterations | Vinsome, Saad, van der Vorst, Sorensen |
| 1977 | Preconditioned matrix iterations | van der Vorst, Meijerink |
| 1977 | MATLAB | Moler |
| 1977 | IEEE arithmetic | Kahan |
| 1982 | Wavelets | Morlet, Grossmann, Meyer, Daubechies |
| 1984 | interior-point methods | Fiacco, McCormick, Karmarkar, Megiddo |
| 1987 | Fast multipole method | Rokhlin, Greengard |
| 1991 | Automatic differentiation | Iri, Bischof, Carle, Griewank |





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<http://w3.bretagne.ens-cachan.fr/math/people/yannick.privat>
- [Emmanuel Trélat](http://www.ljll.math.upmc.fr/trelat/), Université Pierre et Marie Curie and IUF, France,
<http://www.ljll.math.upmc.fr/trelat/>